Trigonometric Integration

Think carefully when integrating trigonmetric functions.
Different *problems* will need different *tricks*.

Some Problems are Easy

E.g. Compute $\int \cos(6x)\,dx$

$$= \frac{\sin(6x)}{6} + C$$

Want

$$\frac{d}{dx}\left[\qquad\right] = \cos(6x)$$

Guess

$$\frac{d}{dx}\left[\frac{\sin(6x)}{6}\right] = \frac{\cos(6x)\cdot \cancel{6}}{\cancel{6}}$$

Our Tools are Trig Identities

$$\sin^2(x) + \cos^2(x) = 1$$
$$\tan^2(x) + 1 = \sec^2(x)$$
$$1 + \cot^2(x) = \csc^2(x)$$

pythagorean identities

$$\sin(2x) = 2\sin(x)\cos(x)$$

double ∡ formula

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$
$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

double ∡ formulas for cos
solved for $\cos^2(x)$ and $\sin^2(x)$

E.g. Compute $\int \boxed{\sin^2(3x)}\, dx$ $\Leftarrow$ Too Hard $\rightarrow$ must rewrite the function

Know $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$

$\boxed{\sin^2(3x)} = \boxed{\dfrac{1-\cos(2\cdot 3x)}{2}}$

$$= \int \frac{1 - \cos(6x)}{2}\, dx$$

$$= \int \underbrace{\frac{1}{2} - \frac{1}{2}\cos(6x)}\, dx$$

$$= \frac{1}{2}x - \frac{1}{2}\cdot \underbrace{\frac{\sin(6x)}{6}}_{\text{see previous problem}} + C$$

E.g. Compute $\int \sin^2(x)\cos(x)\,dx$ $= \int (\underbrace{\sin(x)}_{u})^2 \cdot \underbrace{\cos(x)\,dx}_{du}$

u-sub $\underline{u=\sin(x)}$ $\Rightarrow$ $\frac{du}{dx} = \cos(x)$ $\Rightarrow$ $du = \underline{\cos(x)dx}$

$= \int u^2 \cdot du$

$= \frac{u^3}{3} + C$

$= \frac{\sin^3(x)}{3} + C$

E.g. Compute $\int \cos^3(x)\,dx$ $= \int \underbrace{\cos^2(x)} \cdot \cos(x)\,dx$

$$\boxed{\begin{aligned} \cos^2 x + \sin^2 x &= 1 \\ \cos^2 x &= 1 - \sin^2 x \end{aligned}}$$

$$= \int \Big(1 - (\overbrace{\sin x}^{u})^2 \Big) \cdot \overbrace{\cos(x)\,dx}^{du}$$

u-sub. $u = \sin(x) \Rightarrow du = \cos(x)\,dx$

$$= \int (1 - u^2) \cdot du$$

$$= u - \frac{u^3}{3} + C = \sin(x) - \frac{\sin^3(x)}{3} + C$$