Approximating Roots

Stephen Flood

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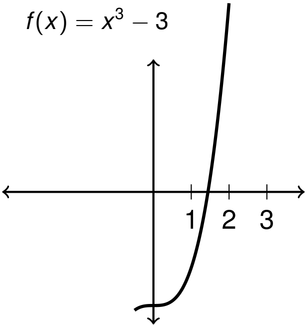
By the intermediate value theorem, we will have

$$a < x = \sqrt[3]{3} < b$$

giving us *upper* and *lower* bounds for the root $x = \sqrt[3]{3}$

$f(x) = x^3 - 3$



x	f(x)

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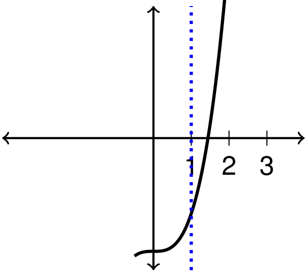
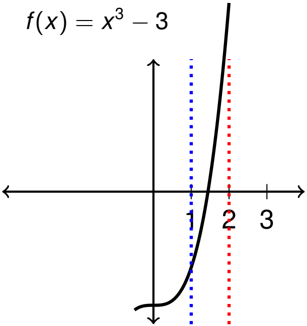


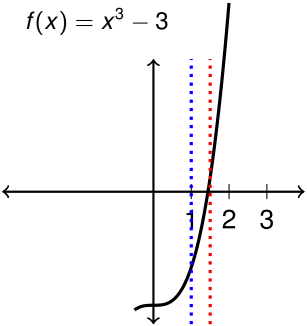
x	f(x)
1	-2



x	f(x)
1	-2
2	5

By the *intermediate value theorem*, there is a root $x = \sqrt[3]{3}$ s.t.

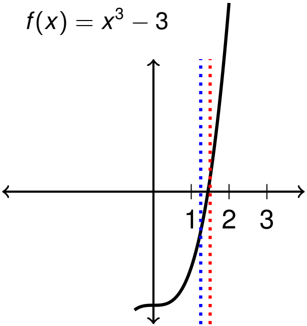
$1 < x < 2$



x	f(x)
1	-2
1.5	0.375
2	5

By the *intermediate value theorem*, there is a root $x = \sqrt[3]{3}$ s.t.

$$1 < x < 1.5$$



x	f(x)
1	-2
1.25	-1.046875
1.5	0.375
2	5

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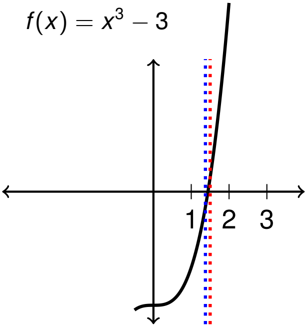
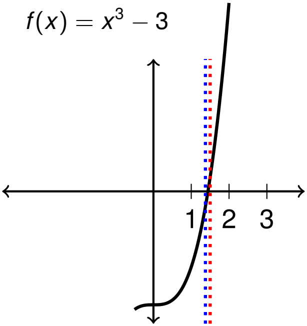
$$1.25 < x < 1.5$$



x	f(x)
1	-2
1.25	-1.046875
1.375	-0.400390625
1.5	0.375
2	5

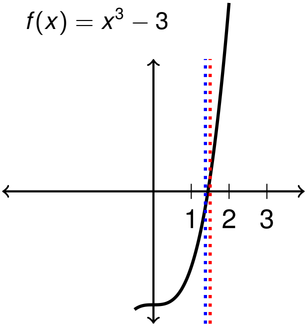
By the *intermediate value theorem*, there is a root $x = \sqrt[3]{3}$ s.t.

$$1.375 < x < 1.5$$



x	f(x)
1	-2
1.25	-1.046875
1.375	-0.400390625
1.5	0.375
2	5

In other words, $\sqrt[3]{3}$ is between 1.375 and 1.5



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1.375	-0.400390625
1.5	0.375
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In fact, $\sqrt[3]{3} = 1.442\ldots$