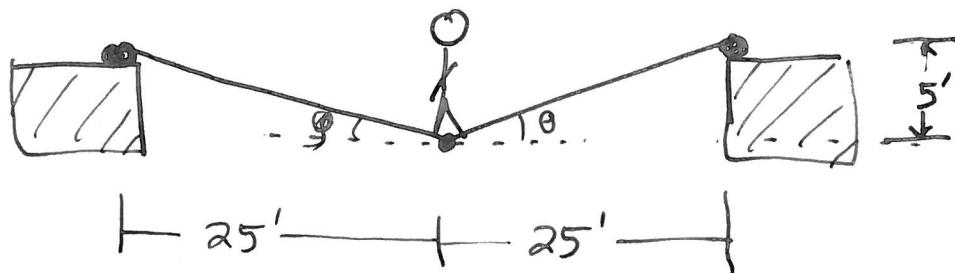


Building a Bridge to Engineering

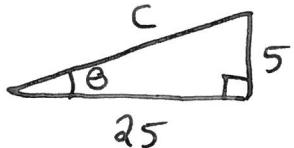
for more details, & more on engineering in general,
see Exploring Engineering, by Kosky et. al.
Chapters 12.1 - 12.6.

warmup: an applied review of Trig.



what is the angle θ from rope to horizontal?

on RHS,

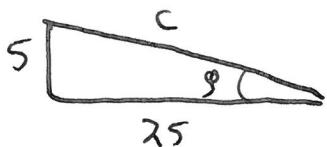


$$\tan(\theta) = \frac{y}{x} = \frac{5}{25} = \frac{1}{5}$$

using a calculator,

$$\theta = \tan^{-1}\left(\frac{1}{5}\right) \approx 11.3^\circ$$

on LHS

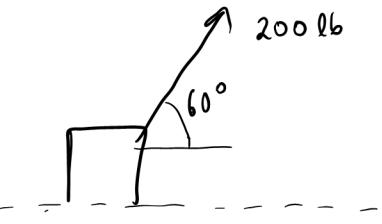


this is the same triangle!
so $\theta = \tan^{-1}\left(\frac{1}{5}\right) \approx 11.3^\circ$

Components of Diagonal Forces

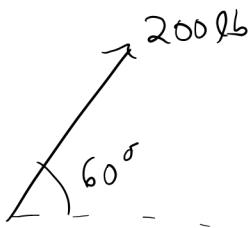
if you pull a box w/ force of 200 lb

at an \angle of 60° from pos x-axis

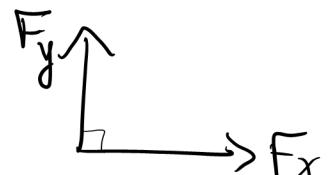


then you are both pulling the box up
AND pulling the box right

Mathematically:



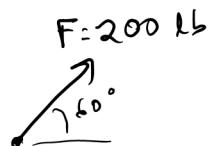
is equal to the sum
of two perpendicular
component forces



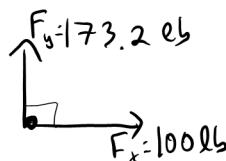
where $F_y = 200 \cdot \sin(60^\circ) \approx 200 \cdot (0.866) \approx 173.2 \text{ lb}$

and $F_x = 200 \cdot \cos(60^\circ) = 200 \cdot \frac{1}{2} = 100 \text{ lb}$

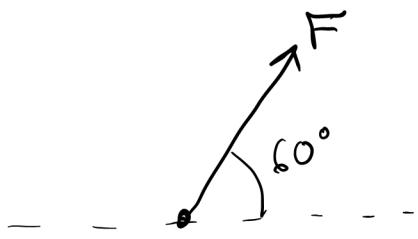
This means that



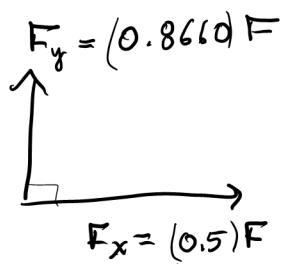
is equivalent to



let the variable F denote the force along the rope

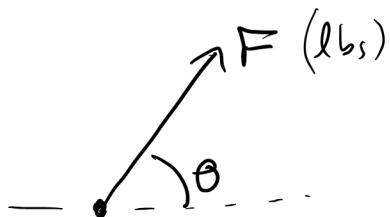


is equivalent to

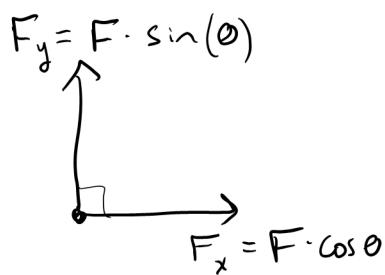


In General

a force of F lbs at θ degrees from positive x -axis

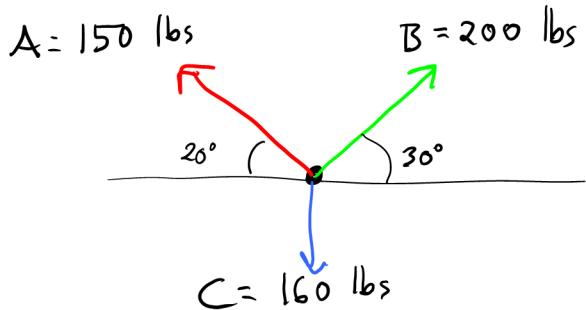


is equal to

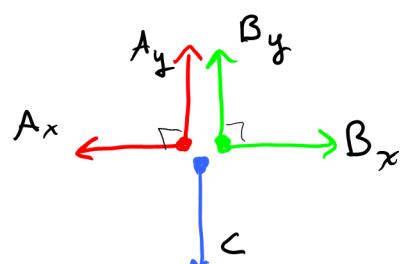


Systems with Multiple Forces

write components
of each forces



is equivalent to

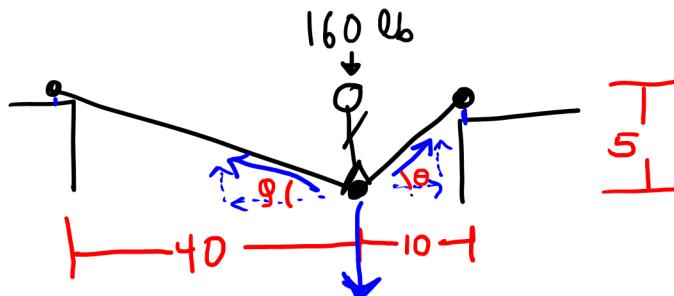


use trigonometry and a calculator to
find coefficients

$$\begin{aligned} B_x &= \cos(30) \cdot B \\ B_y &= \sin(30) \cdot B \\ &\dots \end{aligned}$$

Forces on Rope Bridges

Consider

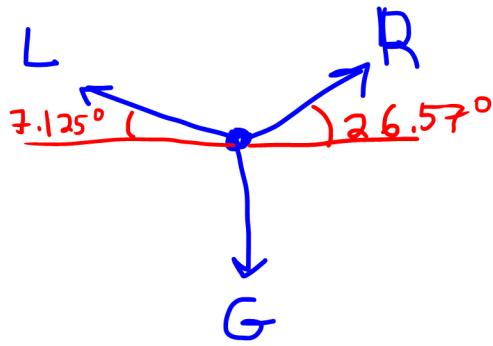


What are the forces on the two halves of the rope?

① Step 1 use trig to compute θ 's.

$$\phi = 7.125^\circ \quad \text{and} \quad \theta = 26.57^\circ$$

② Step 2: Zoom into person & draw a body-free diagram
(free body diagram)



NOTE: G has only an up/down component

find vertical & horizontal components
of L and R

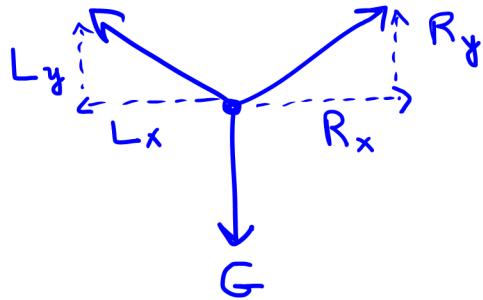
L & R forces have two
components ① up/down part
AND ② left/right part

$$\begin{aligned} (L)_y &= L \cdot \sin(7.125) \\ &= L \cdot \sin(7.125) \\ &\approx L \cdot (0.124) \end{aligned}$$

$$\begin{aligned} (L)_x &= L \cdot \cos(7.125) \\ &\text{total force } x\text{-component} \\ &\approx L \cdot (0.992) \end{aligned}$$

$$\begin{aligned} (R)_y &= R \cdot \sin(26.57) \\ &= R \cdot \sin(26.57) \\ &\approx R \cdot (0.447) \end{aligned}$$

$$\begin{aligned} (R)_x &= R \cdot \cos(26.57) \\ &\approx R \cdot (0.894) \end{aligned}$$



Key idea:

Bridge is not moving
 \Leftrightarrow
 all the forces cancel out.

the bridge is not moving left/right

\Leftrightarrow
 the x-components of the forces cancel out

\Leftrightarrow

$$(L)_x - (R)_x = 0$$

$$0.992 \cdot L - 0.894 \cdot R = 0$$

the bridge is not moving up/down

\Leftrightarrow

the y-components of forces all cancel

\Leftrightarrow

$$\underbrace{(L)_y + (R)_y}_{\text{up}} = \underbrace{G}_{\text{down}}$$

\Leftrightarrow

$$0.124 \cdot L + 0.447 \cdot R = 160$$

The Bridge is not moving at all

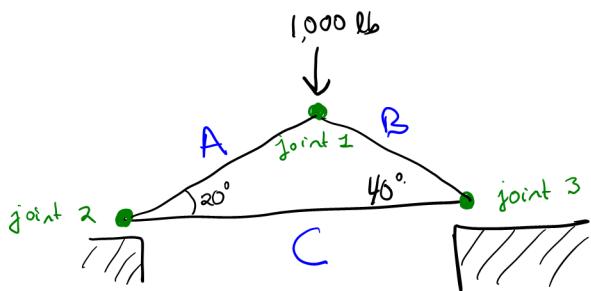
solve

$$\Leftrightarrow \begin{cases} 0.992 L - 0.894 R = 0 \\ 0.124 L + 0.447 R = 160 \end{cases}$$

$$\begin{bmatrix} 0.992 & -0.894 & | & 0 \\ 0.124 & 0.447 & | & 160 \end{bmatrix}$$

Study Rigid Bridges using Method of Joints

Eg: Given a Rigid Bridge

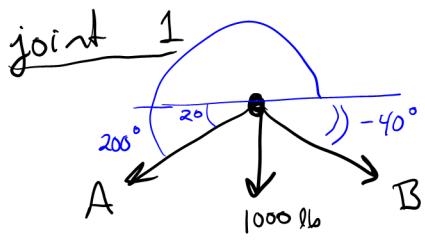


the bridge is stable (not moving)
 no joint is moving
 ⇔
 the forces on each joint sum to zero.

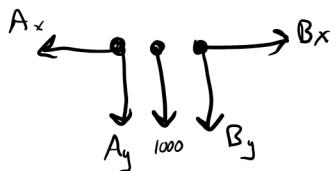
① Label Forces on each member

(optional key): $A = \text{force on top left Beam}$
 $B = \text{force on top right Beam}$
 $C = \text{force on bottom Beam}$

② set up a system of equations for each joint



which is equivalent to



which yields the system

(the x components)
 add to zero

(the y components)
 add to the explicit force on the joint

$$\begin{cases} \cos(20^\circ)A + \cos(-40^\circ)B = 0 \\ \sin(20^\circ)A + \sin(-40^\circ)B = 1000 \end{cases}$$

(each numerical force down on a joint goes on the RHS of the y-component eqn)

NOTE:
 measure all x's from positive x-axis

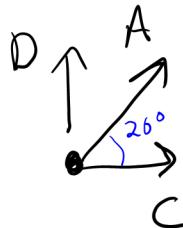
the system simplifies to

$$\begin{cases} -0.9397 \cdot A + 0.7660 \cdot B = 0 \\ -0.3420 \cdot A - 0.6428 \cdot B = 1000 \end{cases}$$

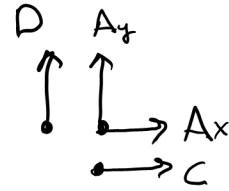
the weight on joint 1 once

For each ground joint, add an additional force representing "Ground pushing up" on the joint.

joint 2



is equivalent to



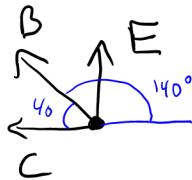
key: D is force of ground up on left joint

which yields the system

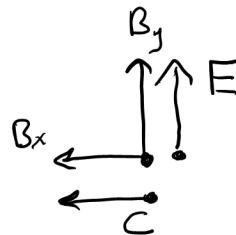
$$\begin{aligned}\sum F_x &= 0 & \left\{ \begin{array}{l} \cos(20^\circ) A + C = 0 \\ \sin(20^\circ) A + D = 0 \end{array} \right. \\ \sum F_y &= 0\end{aligned}$$

(there is no explicit y-force on this joint)

joint 3



is equivalent to



which yields the system

$$\begin{aligned}\sum F_x &= 0 & \left\{ \begin{array}{l} \cos(140^\circ) B + C = 0 \\ \sin(140^\circ) B + E = 0 \end{array} \right. \\ \sum F_y &= 0\end{aligned}$$

Putting these systems together, we get a matrix

$$\left[\begin{array}{ccccc|c} A & B & C & D & E & \text{(numerical forces)} \\ -0.9397 & 0.7660 & 0 & 0 & 0 & 0 \\ -0.3420 & -0.6428 & 0 & 0 & 0 & 1000 \end{array} \right]$$

joint 1: x
joint 1: y
joint 2: x
joint 2: y
joint 3: x
joint 3: y