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Exploring vs Proving Implications

- To **explore** an implication, try picking a variety of different #'s for the variables. Ask yourself "is the implication true for the #'s that you have chosen?
- To **prove** an implication, you must use **arbitrary variables** in a proof.

  To prove $P(x, y) \to Q(x, y)$
  - Suppose that $P(x, y)$ is true.
  - Write down what $P(x, y)$ asserts about x and y.
  - Perform a computation.
  - Try to show that the condition defining $Q(x, y)$ is true.
  - Conclude that $Q(x, y)$ is true.

1. Consider the sentence "If x and y are odd integers, then $x + y$ is even integer."
   (a) Write this as a sentence of logic. Explicitly state all relevant quantifiers.

   (b) **Explore** the sentence the using the selections $\langle x, y\rangle = \langle 3, 5\rangle$, $\langle x, y\rangle = \langle -7, 11\rangle$, and $\langle x, y\rangle = \langle 6, 3\rangle$. Is the sentence true for these examples?

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(c) **Prove** the sentence "If x and y are odd integers, then $x + y$ is an even integer."

To help you get started, I have included "training wheels" in italics.
In the future, you should **write these steps down yourself**.

1. (Assume Hypothesis:)

Suppose that _____

2. (Apply Definition of Hypothesis:)

Then $x =$

and $y =$

Therefore

3. (Work Toward Definition of Conclusion)

And so $x + y =$

4. (Draw Conclusion)

Therefore _____

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2. Consider the statement

   "The difference between *any* odd integer and *any* even integer is an odd integer."

   (a) Rewrite the statement as a sentence of logic.

   (b) Explore the sentence.

   (c) Prove the sentence.

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3. Consider the statement

   "The square of any odd integer is an odd integer."

   (a) Rewrite the statement as a sentence of logic.

   (b) Explore the sentence.

   (c) Prove the sentence.

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4. Consider the statement

   "If x is divisible by 6, then x is even."

   (a) Rewrite the statement as a sentence of logic.

   (b) Explore the sentence.

   (c) Prove the sentence.

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5. Consider the statement

"Let $a, b, c \in \mathbb{Z}$. If $a|b$ and $b|c$, then $a|c$."

(a) Rewrite the statement as a sentence of logic.

(b) Explore the sentence.

(c) Prove the sentence.

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6. Consider the statement

"Let $a, b, c \in \mathbb{Z}$. If $a|b$, then $a|bc$."

(a) Rewrite the statement as a sentence of logic.

(b) Explore the sentence.

(c) Prove the sentence.

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Important, and Assumed, Properties of Integers

The proofs we have given assume three important properties about integers.

1. Negating an integer results in an integer.
2. Adding and subtracting integers results in an integer.
3. Multiplying integers results in an integer.

Termonology: We say $\mathbb{Z}$ is *closed* under negating, adding, subtracting, and multiplying because you cannot escape $\mathbb{Z}$ by performing these operations on its elements.

We will not attempt to prove these properties of integers. In fact, they are so intertwined with the definition of the integers, that to understand what they are saying, and to understand what the integers are, is just the same as to see that they are true.

In the language of analytic philosophy, these are "properly basic beliefs."