Name: _____ Section: _____

Statements about Relations

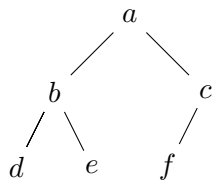
- **Define:** A relation R on A is *reflexive* if and only if $(\forall x \in A)\Big[\ \langle x, x\rangle \in R\ \Big]$

- **Define:** A relation R on A is *symmetric* if and only if $(\forall x, y \in A)\Big[$ if $\langle x, y\rangle \in R$, then $\langle y, x\rangle \in R\ \Big]$

- **Define:** A relation R on A is *transitive* if and only if $(\forall x, y, z \in A)\Big[$ if $\langle x, y\rangle \in R\ \wedge\ \langle y, z\rangle \in R$, then $\langle x, z\rangle \in R\ \Big]$

Describing Relations

A certain simplified family tree T is given on the right. We say that b is a *daughter* of a, because there is a line down from a to b.



1. Define on T a relation C, by $\langle x, y\rangle \in C$ if and only if y is a daughter of x.

   List the elements of C in roster notation. Is C symmetric? Is C reflexive? Is C transitive?

2. Define on T a relation S, by $\langle x, y\rangle \in S$ if and only if x and y have the same parents.

   List the elements of S in roster notation. Is S symmetric? Is S reflexive? Is S transitive?

3. Define on T a relation D, by $\langle x, y\rangle \in D$ if and only if y is a descendent of x.

   List the elements of D in roster notation. Is D symmetric? Is D reflexive? Is D transitive?

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Proofs about Relations

For each of the following, either give a *general proof* that the given relation has the specified property, or give a *counterexample* that shows that it does not have the property.

1. On $\mathbb{N}$, define a relation R by $\langle n, m\rangle \in R$ if and only if $n + m$ is a multiple of 3
   (a) Is the relation reflexive? Give a proof or counterexample.

   (b) Is the relation symmetric? Give a proof or counterexample.

   (c) Is the relation transitive? Give a proof or counterexample.

2. On the set $\mathbb{N}$, define a relation S by $\langle n, m\rangle \in R$ if and only if $n = m^2$.
   (a) Is the relation reflexive? Give a proof or counterexample.

   (b) Is the relation symmetric? Give a proof or counterexample.

   (c) Is the relation transitive? Give a proof or counterexample.

3. On the set of all *logical propositions*, define a relation I by $\langle P, Q\rangle \in I$ if and only if $P \equiv Q$.
   (a) Is the relation reflexive? Give a proof or counterexample.

   (b) Is the relation symmetric? Give a proof or counterexample.

   (c) Is the relation transitive? Give a proof or counterexample.