

Name: Solutions

Section: _____

1. Fill in the blank using an *equivalent* statement from *memory*.

(a) $\neg P \vee Q \equiv P \rightarrow Q$

(b) $P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

(c) $P \rightarrow Q \equiv (\neg P) \vee Q$ (give a statement without \rightarrow)

(d) $\neg(\neg P) \equiv P$

(e) $\neg P \rightarrow \neg Q \equiv Q \rightarrow P$ (give a statement with no negations)

(f) P is sufficient for $Q \equiv P \rightarrow Q$ (use only logical operations)

(g) P is necessary for $Q \equiv \neg P \rightarrow \neg Q$
 $\equiv Q \rightarrow P$ (use only logical operations)

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3. Prove that the statement $\neg(P \wedge Q)$ is logically equivalent to $\neg P \vee \neg Q$.

①

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

② they do have same truth value in all rows,

③ Thus they are equivalent.

4. Prove that the statement $P \wedge (Q \vee R)$ is logically equivalent to $(P \wedge Q) \vee (P \wedge R)$.

①

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$	$(P \wedge Q)$	$(P \wedge R)$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

② they do have same truth values in all rows

③ Thus they are equivalent

5. Decide whether or not the following pairs of statements are equivalent.

$(P \wedge \neg Q)$ and $P \rightarrow \neg Q$

Give a proof to justify your answer.

P	Q	$\neg Q$	$P \wedge \neg Q$	$P \rightarrow \neg Q$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

They do not have same truth values in rows 3 & 4

Thus they are not logically equivalent.

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5. Consider the implication: $P \rightarrow Q$

(a) Write the converse symbolically. Is the converse equivalent to the original sentence? If it is not equivalent, provide a counterexample. counterexample:

Converse: $Q \rightarrow P$ | if P is False & Q is True,
not equivalent | then $P \rightarrow Q$ is true
 but $Q \rightarrow P$ is false

(b) Write the contrapositive symbolically. Is the contrapositive equivalent to the original sentence? If it is not equivalent, provide a counterexample.

Contrapositive: $\neg Q \rightarrow \neg P$ | the contrapositive
 is equivalent to $P \rightarrow Q$

$P \rightarrow \neg Q$
 $\neg Q \rightarrow \neg P$

6. For the proposition: If I am not too warm, then the temperature is just right.

(a) Write it in symbolic logic

W : I am too warm $\neg W \rightarrow R$
 R : The temp is just right

(b) Write it as an equivalent "or" sentence symbolically and in English, and simplify.

Recall: $\neg(P \rightarrow \neg Q) \equiv \neg \neg P \vee \neg \neg Q$
 so $\neg(\neg W \rightarrow R) \equiv \neg \neg W \vee R \equiv W \vee R$

English: I am too warm, or the temp is just right

(c) Write its converse symbolically and in English, and simplify.

Converse: $R \rightarrow \neg W$

English: If the temp is right, then I am not warm.

(d) Write its contrapositive symbolically and in English, and simplify.

Contrapositive: $\neg R \rightarrow \neg \neg W \equiv \neg R \rightarrow W$

English: if the temp is not right, then I am too warm.

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8. Write the following statement as an "or" statement in symbols *and* in English.

"If you give an inch, they'll take a mile."

P: you give an inch

Q: they take a mile

$$\boxed{P} \rightarrow Q \equiv \neg (\boxed{P} \vee Q)$$

in english:

Do not give an inch,
or they'll take a mile.

9. Write the following statement as an "if-then" statement in symbols *and* in English.

"Unite or die."

P: ~~we~~ we unite

Q: ~~we~~ we die

$$\begin{aligned} P \vee Q &\equiv \neg(\neg P) \vee Q \\ &\equiv \neg P \rightarrow Q. \end{aligned}$$

in english:

If β we do not unite,
then we will die.