Written Homework #4. Math 141 Due April 3, 2013

Name: Key Section: ____________

1. Find and draw the interval of convergence of the following power series.

$$\sum_{n=0}^{\infty} \frac{(x+2)^n}{n^2}$$

(Notice: $x+2 = x-(-2)$ $\Rightarrow$ power series is centered at -2)

Ratio test: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(x+2)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(x+2)^n} \right| = \lim_{n\to\infty} \left(|x+2| \cdot \underbrace{\frac{n^2}{(n+1)^2}}_{\downarrow\, 1} \right) = |x+2|$

power series converges if $|x+2| < 1$

$-1 < (x+2) < 1$

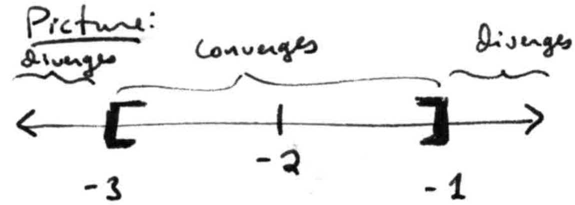
$-3 < x < -1$

Diverges if $|x+2| > 1$

if $x+2 > 1 \Rightarrow x > -1$

or $x+2 < -1 \Rightarrow x < -3$

Picture:



Endpoints: $(x+2) = 1 \Rightarrow x = -1$

$$\sum_{n=0}^{\infty} \frac{(-1+2)^n}{n^2} = \sum_{n=0}^{\infty} \frac{1}{n^2}$$

Converges: p-series $p > 2$

$(x+2) = -1 \Rightarrow x = -3$

$$\sum_{n=0}^{\infty} \frac{(-3+2)^n}{n^2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2}$$

converges: alternating series test

2. Find and draw the interval of convergence of the following power series.

$$\sum_{n=0}^{\infty} n^n (x-1)^n$$

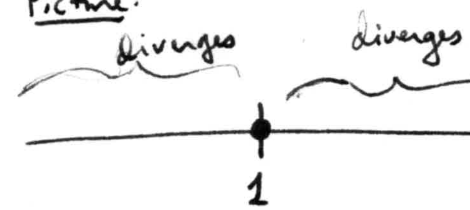
Notice: power series is centered at 1

Root test: $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \left(\left| n^n (x-1)^n \right| \right)^{\frac{1}{n}}$

$= \lim_{n\to\infty} \left(\left| n \cdot (x-1) \right|^n \right)^{\frac{1}{n}}$

$= \lim_{n\to\infty} \underbrace{n}_{\text{goes to infinity}} \cdot \underbrace{|x-1|}_{\text{is constant}}$

Picture:



converges $\Leftrightarrow x = 1$

Notice if $x = 1 \Rightarrow (x-1) = 0$ is constant

and $\lim_{n\to\infty} n \cdot 0 = \lim_{n\to\infty} 0 = 0$

$\Rightarrow$ converges if $x = 1$

if $x \neq 1 \Rightarrow \lim_{n\to\infty} n \cdot |x-1| = \infty > 1$

$\Rightarrow$ diverges if $x \neq 1$

Name: ______________________ Section: ______________________

3. Find and draw the interval of convergence of the following power series.

$$\sum_{n=0}^{\infty} n^2(x-1)^n$$

Notice: centered at $x = 1$

Ratio test: $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \left|\lim\limits_{n\to\infty}\left|\dfrac{(n+1)^2\cdot(x-1)^{n+1}}{n^2\cdot(x-1)^n}\right|\right| = \left|\lim\limits_{n\to\infty}\underbrace{\dfrac{(n+1)^2}{n^2}}_{\downarrow\,1}\cdot|x-1|\right| = |x-1|$

Converges if $|x-1|<1$

if $-1<x-1<1$

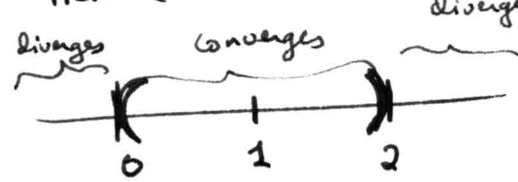
if $0<x<2$

Diverges if $|x-1|>1$

if $x-1>1 \Rightarrow x>2$

$x-1<-1 \Rightarrow x<0$

Picture



Endpoints: if $x-1=1 \Rightarrow x=2$

$\sum\limits_{n=0}^{\infty} n^2\cdot(2-1)^n = \sum\limits_{n=0}^{\infty} n^2$ diverges, divergence test

if $x-1=-1 \Rightarrow x=0$

$\sum\limits_{n=0}^{\infty} n^2(0-1)^n = \sum n^2(-1)^n$ diverges, divergence test

4. Re-index the series $\sum\limits_{n=1}^{\infty}\dfrac{2^n}{(n-1)!}x^{n-1}$ to begin with $n=0$.

(let $m=n-1$)

$$= \sum_{m=0}^{\infty}\frac{2^{m+1}}{m!}x^m = \sum_{n=0}^{\infty}\frac{2^{n+1}}{n!}x^n$$

rename the variable

5. Re-index the series $\sum\limits_{n=0}^{\infty}\dfrac{2^{n+1}}{2n+3}x^{2n+2}$ to begin with $n=1$.

(let $m=n+1$) rename the variable

$$= \sum_{n=0}^{\infty}\frac{2^{n+1}}{2(n+1)+1}x^{2(n+1)} = \sum_{m=1}^{\infty}\frac{2^m}{2m+1}x^{2m} = \sum_{n=1}^{\infty}\frac{2^n}{2n+1}x^{2n}$$

6. Re-index the series $\sum\limits_{n=1}^{\infty} a\left(\dfrac{x}{r}\right)^{n-1}$ to begin with $n=0$.

(let $m=n-1$) (rename)

$$= \sum_{m=0}^{\infty} a\left(\frac{x}{r}\right)^m = \sum_{n=0}^{\infty} a\left(\frac{x}{r}\right)^n$$

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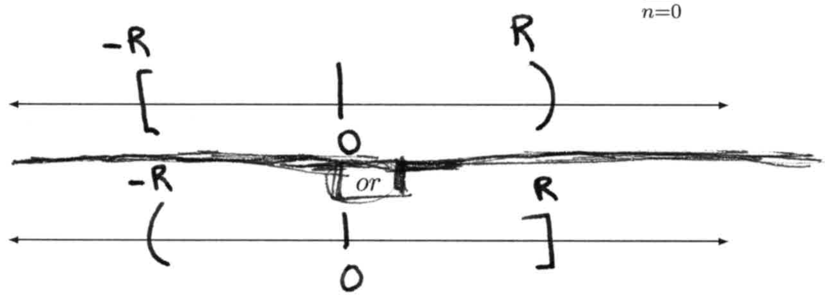
7. ***Optional* Synthesis Problem:**

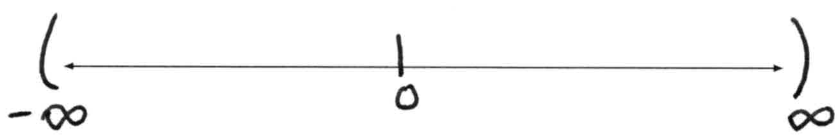
What does the interval of convergence of $\sum_{n=0}^{\infty} c_n x^n$ look like?

The endpoint behavior of the interval of convergence of a power series depends largely on the constant c_n of the x^n term. The trick in this problem is to combine the examples of power series convergence intervals you've seen with your understanding of series comparison tests.

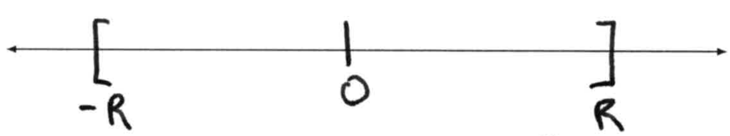
(a) If $\sum_{n=0}^{\infty} c_n$ is alternating and converges conditionally, the interval of $\sum_{n=0}^{\infty} c_n x^n$ is either



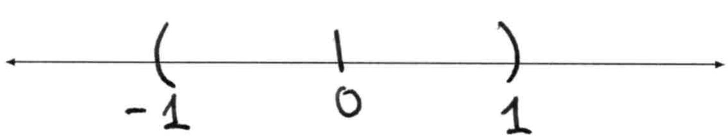
(b) If $\lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right| = 0$, the interval of $\sum_{n=0}^{\infty} c_n x^n$ is



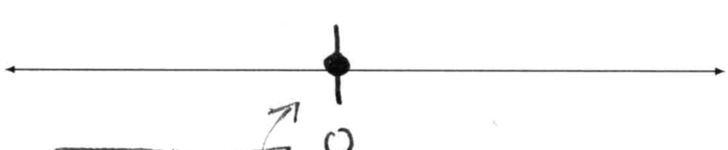
(c) If $\sum_{n=0}^{\infty} c_n$ converges absolutely *and* $\sum_{n=0}^{\infty} c_n x^n$ has a finite radius of convergence R,



(d) If c_n is a polynomial that goes to infinity, the interval of $\sum_{n=0}^{\infty} c_n x^n$ is



(e) If c_n goes to infinity faster than every geometric sequence, the interval of $\sum_{n=0}^{\infty} c_n x^n$ is



$\boxed{\text{only converges at } x = 0}$