Written Homework #2. Math 141 Due March 15

Name: Key Section:

Directions:

Please read each problem *carefully*. There are many different problems that look very similar! Pay close attention to whether you are working with a series, a sequence, or a function.

You will have around 3-4 minutes for these types of problems on the exam. Try completing the homework in at most 30 minutes. Then, on a clean copy/page, work through each problem very carefully. How do your answers compare?

Don't read page 4 until you are happy with your answers and work. It points out some common mistakes you might have made, giving you one last chance to correct them.

The goal here is to practice checking your work and catching tricky problems on an exam.

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1. Does the series $\sum_{n=0}^{\infty} 5 \cdot \frac{(-1)^n \cdot 3^{n+1}}{4^n}$ converge? If so, what does it equal?

Notice: geometric, But starts with $n=0$

$$= \sum_{n=0}^{\infty} 5 \cdot \left(\frac{-3}{4}\right)^n \cdot 3$$

$$= \sum_{n=0}^{\infty} 15 \cdot \left(\frac{-3}{4}\right)^n$$

Notice: the common ratio $= r = \frac{-3}{4}$

Notice the first term $= a = a_0 = 15$

Geometric series with $|r| = \frac{3}{4} < 1 \Rightarrow$ converges

$$= \frac{a}{1-r} = \frac{15}{1+\frac{3}{4}} \cdot \frac{4}{4} = \frac{60}{4+3} = \boxed{\frac{60}{7}}$$

the series converges to

2. Does the sequence $\left\{\frac{\sin(n^2) - n^2}{3n^2}\right\}$ converge? If so, what does it converge to?

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sin(n^2) - n^2}{3n^2} = \lim_{n\to\infty} \frac{\cancel{n^2}\left(\frac{\sin(n^2)}{n^2} - 1\right)}{\cancel{n^2} \cdot 3}$$

$\frac{\sin(n^2)}{n^2} \to 0$

$$= \frac{-1}{3}$$

$\boxed{\text{the } (n^{th} \text{ term of the}) \text{ sequence converges to } \frac{-1}{3}}$

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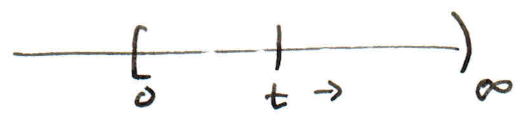
3. Does the series $\sum_{n=1}^{\infty} \frac{\sin(n^2) - n^2}{3n^2}$ converge? Justify your answer.

In problem #2, we saw that the n^{th} term of this sum Does <u>NOT</u> go to 0

$\boxed{\Rightarrow \text{the series diverges by the divergence test}}$

4. Determine if the following integral converges. **You must show *all* work for credit.**

$$\int_0^{\infty} xe^{-x}\,dx$$



$$= \lim_{t\to\infty} \int_0^t x \cdot e^{-x}\,dx$$

<u>Integrate by parts:</u> $\left(\begin{array}{ll} u = x & v = -e^{-x} \\ du = dx & dv = e^{-x}\,dx \end{array} \right)$

$$= \lim_{t\to\infty} \left[-xe^{-x} + \int_0^t +e^{-x}\,dx \right]$$

$$\int e^{-x}\,dx = -e^{-x}$$

$$= \lim_{t\to\infty} \left[-xe^{-x} - e^{-x} \right]_0^t = \lim_{t\to\infty} \left[-\underset{\downarrow \atop \infty}{t} \cdot \underset{\downarrow \atop 0}{e^{-t}} - \underset{\downarrow \atop 0}{e^{-t}} \right] - \underbrace{[0 - \underset{1}{\overset{}{e^{0}}}]}_{+1}$$

$$\boxed{\begin{array}{l} \text{compute} \\ \lim_{t\to\infty} -t \cdot e^{-t} = \lim_{t\to\infty} \frac{-t}{e^{t}} \begin{array}{l} \to \infty \\ \to \infty \end{array} \overset{H}{=} \lim_{t\to\infty} \frac{-1}{e^{t}} = 0 \end{array}}$$

$$= [0 - 0] - [0 - 1] = 1 \leftarrow \boxed{\text{the integral converges to } 1}$$

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geometric!

5. Does the series $\sum_{n=1}^{\infty}\left[\frac{1}{e^n}+\left(e^{\frac{1}{n}}-e^{\frac{1}{n+1}}\right)\right]$ converge? If so, what does it converge to?

telescoping!

You must show *all* work for credit.

$$\sum_{n=1}^{\infty}\frac{1}{e^n}+\sum_{n=1}^{\infty}\left(e^{\frac{1}{n}}-e^{\frac{1}{n+1}}\right)$$

$$=\sum_{n=1}^{\infty}\frac{1}{e}\left(\frac{1}{e}\right)^{n-1}+\lim_{n\to\infty}S_n$$

(geometric with $|r|<1$)

simplify S_n

$$S_n = e^{\frac{1}{1}}-\cancel{e^{\frac{1}{2}}} + \cancel{e^{\frac{1}{2}}}-\cancel{e^{\frac{1}{3}}} + \cancel{e^{\frac{1}{3}}}-\cancel{e^{\frac{1}{4}}} + \dots + \cancel{e^{\frac{1}{n-1}}}-\cancel{e^{\frac{1}{n}}} + \cancel{e^{\frac{1}{n}}}-e^{\frac{1}{n+1}}$$

$$S_n = e^1 - e^{\frac{1}{n+1}}$$

$$=\frac{a}{1-r}+\lim_{n\to\infty}\left(e-e^{\frac{1}{n+1}}\right)$$

$\frac{1}{n+1}\to 0$, $e^{\frac{1}{n+1}}\to 1$

$$=\frac{\frac{1}{e}}{1-\frac{1}{e}}\cdot\frac{e}{e}+(e-1)$$

$$\boxed{=\frac{1}{e-1}+e-1}$$

the series converges to this!

6. Let $a_n = 5\cdot\frac{(-1)^n\cdot 3^{n+1}}{4^n}$. Does the sequence $\{a_n\}$ converge? If so, what does it converge to?

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} 5\cdot\left(\underbrace{\frac{-3}{4}}_{\text{less than 1 in absolute value}}\right)^{n}\cdot 3 = 0$$

goes to ∞

the (n^{th} term of the) sequence converges to 0

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Spoiler Alert: Some of these problems have a tricky step. Try to solve them completely *without* looking at these hints. Once you are happy with your work, check below to see if you made some common mistakes.

7. Carefully check your work! Here are some small errors you might have made:
   (a) The sum in #1 starts with $n = 0$. Did you select the correct first term?
   (b) Did you carefully write out the computation of the limit in #4?
   (c) Problem #5 is tricky. Did you split it up into two different questions using series arithmetic? Did you show all work for each sub-question?