Name: ______________________ Section: ______________________

1. Let $f(x) = \left(1 + \frac{2}{x}\right)^x$. Does $\lim_{x\to\infty} f(x)$ diverge? If it converges, what does it converge to?

$$\lim_{x\to\infty} \Big(\underbrace{1+\frac{2}{x}}_{\to 1}\Big)^{x \to \infty} = \lim_{x\to\infty} e^{\overbrace{x}^{\to\infty}\cdot \overbrace{\ln\left(1+\frac{2}{x}\right)}^{\to 1,\ \to 0}}$$

$$\boxed{\begin{aligned} \lim_{x\to\infty} \underbrace{x}_{\infty}\cdot\underbrace{\ln\left(1+\frac{2}{x}\right)}_{0} &= \lim_{x\to\infty} \frac{\ln\left(1+\frac{2}{x}\right) \to 0}{\frac{1}{x} \to 0} \\ &\overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{1+\frac{2}{x}}\cdot\left(\frac{-2}{x^2}\right)}{\frac{-1}{x^2}} = \lim_{x\to\infty}\left(\frac{1}{1+\frac{2}{x}}\right)\cdot 2 = 2 \end{aligned}}$$

$$= e^2$$

2. Let $f(x) = \left(\frac{x}{x+3}\right)^x$. Does $\lim_{x\to\infty} f(x)$ diverge? If it converges, what does it converge to?

$$\lim_{x\to\infty} \Big(\underbrace{\frac{x}{x+3}}_{\to 1}\Big)^{x \to\infty} = \lim_{x\to\infty} e^{x\cdot\ln\left(\frac{x}{x+3}\right)} = e^{-3}$$

$$\boxed{\begin{aligned} \lim_{x\to\infty} \underbrace{x}_{\infty}\cdot\underbrace{\ln\left(\frac{x}{x+3}\right)}_{0} &= \lim_{x\to\infty} \frac{\ln\left(\frac{x}{x+3}\right) \to 0}{\frac{1}{x} \to 0} \\ &\overset{H}{=} \lim_{x\to\infty} \frac{\frac{1}{\left(\frac{x+3}{x}\right)}\cdot\left(\frac{(x+3)-x}{(x+3)^2}\right)}{\left(-\frac{1}{x^2}\right)} \\ &= \lim_{x\to\infty} \frac{(\cancel{x}+3-\cancel{x})}{x(x+3)}\cdot\left(\frac{x^2}{-1}\right) = \lim_{x\to\infty} \frac{3\cdot(-x^2)}{x^2+3x} \\ &= \lim_{x\to\infty} \frac{\cancel{x^2}}{\cancel{x^2}}\left(\frac{-3}{1+\frac{3}{x}}\right) = -3 \end{aligned}}$$

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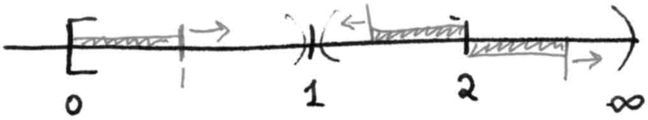
3. Some improper integrals combine *several* cases in surprising ways.

Carefully determine the intervals of continuity and write the integral as the sum of limits of proper integrals. Do not evaluate the individual integrals or limits.



$$\int_0^\infty \frac{1}{(x-1)^2}\,dx$$

$$= \int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^\infty \frac{1}{(x-1)^2}\,dx$$

this is still doubly improper.
pick # between 1 & ∞
& rewrite again

$$= \int_0^1 \frac{1}{(x-1)^2}\,dx + \int_1^2 \frac{1}{(x-1)^2}\,dx + \int_2^\infty \frac{1}{(x-1)^2}\,dx$$

Now turn into limits:

$$= \lim_{t\to 1^-} \int_0^t \frac{1}{(x-1)^2}\,dx + \lim_{t\to 1^+} \int_t^2 \frac{1}{(x-1)^2}\,dx + \lim_{t\to\infty} \int_2^t \frac{1}{(x-1)^2}\,dx$$

4. Use the comparison test to determine if the following integral converges.

$$\int_1^\infty \frac{x\cdot \sin^2 x}{1\,+\,x^3}\,dx$$

compare:

$$\frac{x\cdot \sin^2(x)}{1+x^3} \le \frac{x}{1+x^3} \le \frac{x}{x^3} = \frac{1}{x^2}$$

and check:

$$\boxed{\int_1^\infty \frac{1}{x^2}\,dx} = \lim_{t\to\infty}\int_1^t \frac{1}{x^2}\,dx = \lim_{t\to\infty}\left(\frac{-1}{t} - \frac{-1}{1}\right)$$

converges

$$= -1$$

a larger integral converges ⇒ our integral converges

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5. Write the first four terms of the following sequences. Simplify when possible.

(a) $a_n = \dfrac{2n-1}{2n+2}$

$$\left\{ \frac{2\cdot1-1}{2\cdot1+2}, \frac{2\cdot2-1}{2\cdot2+2}, \frac{2\cdot3-1}{2\cdot3+2}, \frac{2\cdot4-1}{2\cdot4+2}, \dots \right\}$$

$$= \left\{ \frac{1}{4}, \frac{3}{6}, \frac{5}{8}, \frac{7}{10}, \dots \right\}$$

(b) $a_n = \cos\left(\dfrac{n\pi}{2}\right)$

$$\left\{ \cos\left(\tfrac{\pi}{2}\right), \cos(\pi), \cos\left(\tfrac{3\pi}{2}\right), \cos(2\pi), \dots \right\}$$

$$= \{0, -1, 0, 1, \dots\}$$

(c) $a_n = \dfrac{(-1)^{n-1}}{5^n}$

$$\left\{ \frac{(-1)^{1-1}}{5^1}, \frac{(-1)^{2-1}}{5^2}, \frac{(-1)^{3-1}}{5^3}, \frac{(-1)^{4-1}}{5^4}, \dots \right\}$$

$$= \left\{ \frac{1}{5}, \frac{-1}{25}, \frac{1}{125}, \frac{-1}{625}, \dots \right\}$$

(d) $a_1 = 6$ and $a_{n+1} = \dfrac{a_n}{n}$.

$$a_1 = 6, \quad a_2 = \frac{a_1}{1} = \frac{6}{1} = 6, \quad a_3 = \frac{a_2}{2} = \frac{6}{2} = 3, \quad a_4 = \frac{a_3}{3} = \frac{3}{3} = 1$$

$$\{6, 6, 3, 1, \dots\}$$

6. Write the first three terms of the following sequences. Do *not* simplify.

(a) $a_n = \dfrac{1}{(n+1)!}$

$$\left\{ \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \frac{1}{5!}, \dots \right\}$$

$$= \left\{ \frac{1}{2\cdot1}, \frac{1}{3\cdot2\cdot1}, \frac{1}{4\cdot3\cdot2\cdot1}, \frac{1}{5\cdot4\cdot3\cdot2\cdot1}, \dots \right\}$$

(b) $a_n = 7 \cdot \dfrac{2^{n+1}}{3^n}$

$$\left\{ 7\cdot\frac{2^2}{3^1}, 7\cdot\frac{2^3}{3^2}, 7\cdot\frac{2^4}{3^3}, 7\cdot\frac{2^5}{3^4}, \dots \right.$$

Stewart 11.1 #5,6,10

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7. Write a formula for the n^{th} term of each of the following sequences.

(a) $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots\right\}$

$$a_n = \frac{1}{n^{th} \text{ odd\#}}$$

$$\boxed{a_n = \frac{1}{2n-1}}$$

(b) $\left\{1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \ldots\right\}$

$$\boxed{a_n = \frac{1}{3^{n-1}} \cdot (-1)^{n-1} = \left(\frac{-1}{3}\right)^{n-1}}$$

equivalently, $a_n = \frac{(-1)^{n+1}}{3^{n-1}}$

(c) $\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \ldots\right\}$

top = n squared
bottom = 1 + n
sign alternates, begins positive

$$\boxed{a_n = \frac{n^2}{1+n}(-1)^{n-1}}$$

Stewart 11.1 #13,14,17

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8. Write a formula for the n^{th} term of each of the following sequences.

(a) $\left\{ 1, \frac{1}{3}, \frac{1}{3\cdot 4}, \frac{1}{3\cdot 4\cdot 5}, \frac{1}{3\cdot 4\cdot 5\cdot 6}, \dots \right\}$

a_1 a_2 a_3 [redacted] a_4 a_5

$$\underline{\text{bottom}} = (n+1)\cdots 3 = \frac{(n+1)\, n\, (n-1)\cdots 3\cdot 2\cdot 1}{2} = \frac{(n+1)!}{2}$$

So $$\boxed{a_n = \frac{1}{\frac{(n+1)!}{2}} = \frac{2}{(n+1)!}}$$

(b) $\left\{ \frac{1}{2^2}, -\frac{3^3}{4^4}, \frac{5^5}{6^6}, -\frac{7^7}{8^8}, \dots \right\}$

a_1 a_2 a_3 a_4

top = $(n^{th} \text{ odd})^{n^{th} \text{ odd}}$

bottom = $(n^{th} \text{ even})^{n^{th} \text{ even}}$

sign alternates, starts positive

$$\boxed{a_n = \frac{(2n-1)^{(2n-1)}}{(2n)^{2n}} \cdot (-1)^{n+1}}$$

(c) $\left\{ \frac{1}{2\cdot 1}, \frac{2^2}{2\cdot 2\cdot 1}, \frac{3^2}{2\cdot 3\cdot 2\cdot 1}, \frac{4^2}{2\cdot 4\cdot 3\cdot 2\cdot 1}, \dots \right\}$

a_1 a_2 a_3 a_4

top = n^2

bottom = $2\cdot(n!)$

$$a_n = \frac{n^2}{2(n!)}$$

Danger: $(2n)!$ is VERY DIFFERENT

correct use of $(\cdot)$ is important.