Math 141 Quiz #4 April 23, 2013

Name: Key Section: ____________

Question	Answer
1	B
2	D
3	A
4	E
5	B
6	B

There are 6 multiple choice problems, and 2 short answer problems. You have 30 minutes to complete the quiz.

Please mark all multiple choice answers in the box provided.

For the short answer, **show all work**.

1. Suppose that f is some invertible function where $f(3) = 5$, $\boxed{f(4) = 3}$, $f'(3) = 7$ and $\boxed{f'(4) = 11}$. What is $(f^{-1})'(3)$?

   (a) 5

   $\boxed{\text{(b) } \frac{1}{11}}$

   (c) $\frac{1}{5}$

   (d) 3

   (e) $\frac{1}{7}$

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(4)} = \frac{1}{11}$$

$$f^{-1}(3) = x = 4$$

$$\Leftrightarrow$$

$$3 = f(x) = f(4)$$

2. Find the derivative of $f(x) = x^{\tan^{-1}(x)}$

   (a) $x^{\tan^{-1}(x)}\left(\frac{1}{1+x^2} + \frac{1}{x}\right)$

   (b) $x^{\tan^{-1}(x)}\left(\tan^{-1}(x) + \frac{1}{x(1+x^2)}\right)$

   (c) $x^{\tan^{-1}(x)}\left(\frac{x}{\tan^{-1}(x)} + \frac{\ln(x)}{(1+x^2)}\right)$

   $\boxed{\text{(d) } x^{\tan^{-1}(x)}\left(\frac{\ln(x)}{1+x^2} + \frac{\tan^{-1}(x)}{x}\right)}$

   (e) $x^{\tan^{-1}(x)}\left(\frac{1}{x} + \frac{\ln(x)}{1+x^2}\right)$

$$y = x^{\tan^{-1}(x)}$$

$$\ln(y) = \ln\left(x^{\tan^{-1}(x)}\right) = \tan^{-1}(x) \cdot \ln(x)$$

so

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{1+x^2} \cdot \ln(x) + \tan^{-1}(x) \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = x^{\tan^{-1}(x)}\left(\frac{\ln(x)}{1+x^2} + \frac{\tan^{-1}(x)}{x}\right)$$

3. Evaluate the following: $\lim\limits_{x \to 0^+} x \cdot \ln\left(\frac{1}{x}\right)$

   $\boxed{\text{(a) } 0}$

   (b) 1

   (c) 2

   (d) e

   (e) Diverges

$$\underbrace{x}_{\to 0} \cdot \underbrace{\ln\left(\tfrac{1}{x}\right)}_{\to \infty}, \quad \infty$$

$$= \lim_{x \to 0^+} \frac{\ln\left(\frac{1}{x}\right) \to \infty}{\frac{1}{x} \to \infty}$$

$$\overset{H}{=} \lim_{x \to 0^+} \frac{\frac{1}{\left(\frac{1}{x}\right)} \cdot \cancel{\left(\frac{-1}{x^2}\right)}}{\cancel{\left(\frac{-1}{x^2}\right)}} = \lim_{x \to 0^+} \frac{1}{\frac{1}{x}}$$

$$= \lim_{x \to 0^+} x = 0$$

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4. Does the integral $\int_0^1 \frac{1}{x^7}\,dx$ converge? If so, what does it converge to?

   (a) 0

   (b) $\frac{1}{8}$

   (c) 1

   (d) 6

   $\boxed{\text{(e) Diverges}}$

$$= \lim_{t \to 0^+} \int_t^1 \frac{1}{x^7}\,dx = \lim_{t \to 0^+} \int_t^1 x^{-7}\,dx$$

$$= \lim_{t \to 0^+} \left[\frac{x^{-6}}{-6} \right]_t^1 = \lim_{t \to 0^+} \left[\frac{1}{-6} - \frac{1}{-6 \cdot t^6} \right].$$

as $t \to 0$ $\to \infty$

5. Evaluate the following: $\lim_{x \to \infty} e^{-(1/x^2)}$

   (a) 0

   $\boxed{\text{(b) 1}}$

   (c) e

   (d) e^{-2}

   (e) Diverges

as $x \to \infty$

$x^2 \to \infty$

so $\frac{1}{x^2} \to 0$

and $\frac{-1}{x^2} \to 0$

Therefore $e^{-\frac{1}{x^2}} \to e^0 = 1$

6. Determine if the SEQUENCE converges. If it converges, find the value to which it converges.

$$\left\{ \frac{n^3 + n + 7}{n^3 + (-1)^{n-1}} \right\}_{n=1}^{\infty}$$

   (a) 0

   $\boxed{\text{(b) 1}}$

   (c) 7

   (d) 3

   (e) Diverges

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \underbrace{\frac{\cancel{n^3}}{\cancel{n^3}}}_{=1} \cdot \left(\frac{1 + \overbrace{\frac{1}{n^2}}^{\to 0} + \overbrace{\frac{7}{n^3}}^{\to 0}}{1 + \underbrace{\frac{(-1)^{n-1}}{n^3}}_{\to 0}} \right)$$

$$= \frac{1}{1} = 1$$

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power series centered at 3

7. (2 points). Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2 \cdot 3^n}$.

Be sure to determine what happens at the endpoints.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{(n+1)^2\, 3^{n+1}} \cdot \frac{n^2\, 3^n}{(x-3)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(x-3)}{3}\cdot\frac{n^2}{(n+1)^2}\right| = \frac{|x-3|}{3}$$

$$\lim\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x-3|}{3} < 1$$

$\Leftrightarrow$

$|x-3| < 3$

$\Leftrightarrow$

$-3 < (x-3) < 3$

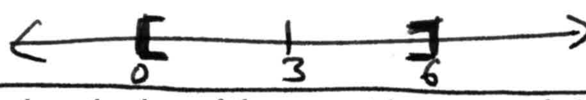
$\Leftrightarrow$

$\boxed{0 < x < 6}$

Plug in endpoints:

$x = 0 \Rightarrow \displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{n^2 (3)^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges (alternating series test)

$x = 6 \Rightarrow \displaystyle\sum_{n=1}^{\infty} \frac{(6-3)^n}{n^2\, 3^n} = \sum_{n=1}^{\infty} \frac{(3)^n}{n^2 \cdot 3^n} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series $p > 2$)



converges for x in $[0,6]$

8. (2 points). Find the Cartesian point where the slope of the parametric curve equals 2.

$x(t) = t + 1$
$y(t) = e^{2t}$

Method 1: (always works)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2 \cdot e^{2t}}{1} = 2 \cdot e^{2t}$$

$$\frac{dy}{dx} = 2 \cdot e^{2t} = 2$$

when $e^{2t} = 1$

so $2t = \ln(1) = 0$

so $t = 0$

$x(0) = 0 + 1 = 1$

$y(0) = e^{2 \cdot 0} = e^0 = 1$

$\boxed{\text{the point is } (1,1)}$

Method 2: (sometimes works)

$x = t + 1 \Rightarrow x - 1 = t$

$y = e^{2t} \Rightarrow y = e^{2(x-1)} = e^{2x-2}$

so $\dfrac{dy}{dx} = \dfrac{d}{dx}\left(e^{2x-2}\right) = 2 \cdot e^{2x-2}$

and $2 \cdot e^{2x-2} = 2$

when $e^{2x-2} = 1$

when $2x - 2 = 0$

when $\boxed{x = 1}$ and

so $y = e^{2x-2} = e^{2\cdot 1 - 2} = e^0 = \boxed{1 = y}$

$\boxed{\text{the point is } (1,1)}$