Math 141 Quiz #4: Thursday, December 6, 2012

Name: Key Section: ____________________

Question	Answer
1	B
2	A
3	D
4	B
5	B
6	D

There are 6 multiple choice problems, and 2 short answer problems. You have 30 minutes to complete the quiz.

Please mark all multiple choice answers in the box provided.

1. Describe the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{(x-3)^n}{n \cdot 2^n}$.

centered at 3

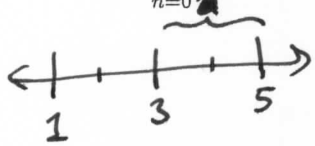
(a) Radius = 3, centered at $a = 2$.

$\boxed{\text{(b)}}$ Radius $= \underline{2}$, centered at $a = \underline{3}$.

(c) Radius = 1, centered at $a = 0$.

(d) Radius = 2, centered at $a = -3$.

(e) Radius = 1, centered at $a = 3$.



$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(x-3)^{n+1}}{(n+1)\,2^{n+1}} \cdot \frac{n \cdot 2^n}{(x-3)^n}\right|$$

$$= \left|\frac{x-3}{2} \cdot \frac{n}{(n+1)}\right|$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{|x-3|}{2} < 1$$

$$\Leftrightarrow \quad \boxed{|x-3| < 2}$$

radius 2

2. Evaluate $\int \frac{x^2}{1+x^2}\,dx$ as a power series centered at $a = 0$.

$\boxed{\text{(a)}}$ $C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+3}}{2n+3}$

(b) $C + \sum_{n=0}^{\infty} (-1)^n x^{2n}$

(c) $C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+3}}{(2n+3)!}$

(d) $C + \sum_{n=0}^{\infty} (-1)^n (2n+3)\, x^{2n+3}$

$$\frac{x^2}{1+x^2} = x^2 \frac{1}{1-(-x^2)} = x^2 \sum_{n=0}^{\infty} (-x^2)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n x^{2n+2}$$

$$\int \frac{x^2}{1+x^2}\,dx = \int \sum_{n=0}^{\infty} (-1)^n x^{2n+2}\,dx$$

$$= C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+3}}{(2n+3)}$$

3. Find the Taylor series for $\frac{d}{dx}e^x$ centered around $x = 0$.

(a) $\sum_{n=1}^{\infty} \frac{x^n}{(n-1)!}$

(b) $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n!}$

(c) $\sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)n!}$

$\boxed{\text{(d)}}$ $\sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!}$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\frac{d}{dx}e^x = \frac{d}{dx}\sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=1}^{\infty} \frac{n \cdot x^{n-1}}{n!} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!}$$

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4. Find the MacLaurin series for $f(x) = x^5 \sin(x^2)$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

(a) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{7n+5}}{(7n+5)!}$

$\boxed{\text{(b)}}$ $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+7}}{(2n+1)!}$

(c) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$

(d) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+7}}{(4n)!}$

$$x^5 \cdot \sin(x^2) = x^5 \cdot \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n+1}}{(2n+1)!}$$

$$= x^5 \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+7}}{(2n+1)!}$$

5. What is the formula for the length of the parametric curve drawn from $t = 0$ to $t = 20$?

~~(a)~~ $\int_{x^{-1}(0)}^{x^{-1}(20)} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$ ← not a formula

$\boxed{\text{(b)}}$ $\int_0^{20} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

~~(c)~~ $\int_{x^{-1}(0)}^{x^{-1}(20)} y(t) \cdot x'(t)\, dt$ ← formula for area under a parametrized function

~~(d)~~ $\int_0^{20} \sqrt{1 + \left(\frac{dy}{dt}\right)^2}\, dt$

↖ not a formula.

6. Find the curve described by parametric equations

$$\begin{cases} x(t) = \cos(t) + 1 & 0 \le t \le 2\pi \\ y(t) = -\sin(t) - 1 \end{cases}$$

(a) $y^2 - (x+1)^2 = -1$

(b) $(y-1)^2 - (x+1)^2 = 1$

(c) $2 = x \cdot y^2$

$\boxed{\text{(d)}}$ $(y+1)^2 + (x-1)^2 = 1$

(e) $y^2 = 2(x-1)^2$

$x = \cos(t) + 1 \Rightarrow \cos(t) = x - 1$

$y = -\sin(t) - 1 \Rightarrow y + 1 = -\sin(t)$

$-(y+1) = \sin(t)$

$(\sin(t))^2 + (\cos(t))^2 = 1$

$\Rightarrow (y+1)^2 + (x-1)^2 = 1$

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7. $\boxed{\text{2 points}}$ Find the points on the following parametric curve where the tangent is horizontal.

$$\begin{cases} x(t) = \ln(t) & \text{for } t > 0 \\ y(t) = t - t^2 \end{cases}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1-2t}{\frac{1}{t}} = t(1-2t)$$

1 pt for computing $\frac{dy}{dx}$

$\frac{dy}{dx} = 0$ <u>if</u> $t = \frac{1}{2}$

<u>so</u> $x(\frac{1}{2}) = \ln(\frac{1}{2}) = -\ln(2)$

$y(\frac{1}{2}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

the curve is horizontal at $(\ln(\frac{1}{2}), \frac{1}{4})$

1 point for finding this point.

O.K. to write it as $(-\ln(2), \frac{1}{4})$.

8. $\boxed{\text{2 points}}$ Find the 3rd degree Taylor Polynomial for $f(x) = \frac{1}{x}$ centered around $x = 2$.

n	$f^{(n)}(x)$	$f^{(n)}(2)$
0	$\frac{1}{x}$	$\frac{1}{2}$
1	$\frac{-1}{x^2}$	$\frac{-1}{4}$
2	$\frac{2}{x^3}$	$\frac{2}{8}$
3	$\frac{-2\cdot 3}{x^4}$	$\frac{-2\cdot 3}{16}$

1 pt for knowing the formula

$$\boxed{T_3(x) = f(2) + \frac{f'(2)}{1!}(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3}$$

$$= \frac{1}{2} + \frac{-1}{4}(x-2) + \frac{2}{8}\cdot\frac{1}{2!}(x-2)^2 + \frac{-2\cdot 3}{16}\cdot\frac{1}{3!}(x-2)^3$$

$$\boxed{T_3(x) = \frac{1}{2} - \frac{(x-2)}{4} + \frac{(x-2)^2}{8} - \frac{(x-2)^3}{16}}$$

1 point for finding $f^{(n)}(2)$ & simplifying