Math 141 Quiz #3: Thursday, April 17, 2013

Name: Key Section: ______

Question	Answer
1	
2	
3	
4	
5	
6	

There are 6 multiple choice problems, and 2 short answer problems. You have 30 minutes to complete the quiz.

Please mark all multiple choice answers in the box provided.

For the short answer, **show all work.**

1. Describe the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{n!\,(x+3)^n}{144^n}$.

$x+3 = x-(-3)$ $\Rightarrow$ centered at $\boxed{a=-3}$

(a) $\boxed{\text{Radius} = 0, \text{ centered at } a = -3.}$

(b) Radius $= 144$, centered at $a = 0$.

(c) Radius $= \frac{1}{144}$, centered at $a = -3$.

(d) Radius $= 1$, centered at $a = 3$.

(e) Radius $= \infty$, centered at $a = -3$.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!\cdot(x+3)^{n+1}}{144^{n+1}}\cdot\frac{144^n}{n!\,(x+3)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(n+1)\cdot(x+3)}{144}\right| = \infty \leftarrow \boxed{\text{IF } x \neq -3}$$

$\Rightarrow$ ratio $= \infty > 0$ when $x \neq -3$

$\boxed{\Rightarrow \text{radius of convergence} = \text{[redacted] } 0}$

2. Evaluate $\int \frac{x}{2-x}\,dx$ as a Maclaurin series (as a power series centered at $a = 0$).

(a) $C + \sum_{n=0}^{\infty} \frac{x^{n+2}}{2^{n+2}}$

(b) $C + \sum_{n=0}^{\infty} \frac{x^{n+1}}{2^{n+1}}$

(c) $\boxed{C + \sum_{n=0}^{\infty} \frac{x^{n+2}}{(n+2)\cdot 2^{n+1}}}$

(d) $C + \sum_{n=0}^{\infty} \frac{x^{n-1}}{n\cdot 2^{n+1}}$

$$\frac{x}{2-x} = \frac{x}{2}\cdot\frac{1}{1-\frac{x}{2}} = \frac{x}{2}\cdot\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n = \sum_{n=0}^{\infty}\frac{x^{n+1}}{2^{n+1}}$$

$$\int\frac{x}{2-x}\,dx = \int\left(\sum_{n=0}^{\infty}\frac{x^{n+1}}{2^{n+1}}\right)dx = C + \sum_{n=0}^{\infty}\left(\int\frac{x^{n+1}}{2^{n+1}}\,dx\right)$$

$$= C + \sum_{n=0}^{\infty}\frac{x^{n+2}}{(n+2)\,2^{n+1}}$$

3. Find the Taylor series for $\frac{d}{dx}\cos(x)$ centered around $x = 0$.

(a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n+2}}{(2n+2)!}$

(b) $\boxed{\sum_{n=1}^{\infty}(-1)^{n}\frac{x^{2n-1}}{(2n-1)!}}$

(c) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n+1}}{(2n-1)!}$

(d) $\sum_{n=1}^{\infty}(-1)^{n}\frac{x^{2n}}{(2n)!}$

$$\frac{d}{dx}\cos(x) = \frac{d}{dx}\left(\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}\right)$$

$$= \sum_{n=1}^{\infty}\frac{d}{dx}\left((-1)^n\frac{x^{2n}}{(2n)!}\right)$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n)!}(2n)\,x^{2n-1\,\text{[redacted]}} = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}\,x^{2n-1}}{(2n-1)!}$$

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4. Find the MacLaurin series for $f(x) = \dfrac{1}{1-x} - e^x$ $= \sum_{n=0}^{\infty} (x)^n - \sum_{n=0}^{\infty} \dfrac{x^n}{n!}$

   (a) $\sum_{n=0}^{\infty} \left(\frac{1}{n!} - \frac{1}{n}\right) x^n$

   (b) $\boxed{\sum_{n=0}^{\infty} \left(1 - \frac{1}{n!}\right) x^n}$

   (c) $\sum_{n=0}^{\infty} (-1)^{n-1} \left(\frac{x}{(2n+1)!} - \frac{1}{(2n)!}\right) x^{2n}$

   (d) $\sum_{n=0}^{\infty} \left(\frac{1}{n!} - 1\right) x^n$

$$= \sum_{n=0}^{\infty} \left[x^n - \frac{x^n}{n!}\right]$$

$$= \sum_{n=0}^{\infty} \left[\left(1 - \frac{1}{n!}\right) x^n\right]$$

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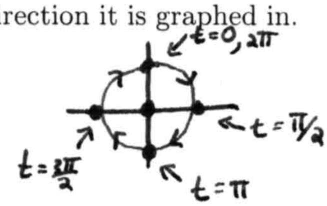
5. In your scrap work (not graded), graph/sketch the **parametric** equation.
   Then (graded) choose the answer that describes the curve, and the direction it is graphed in.

$$\begin{cases} x(t) = \sin(t) & 0 \le t \le 2\pi \\ y(t) = \cos(t) \end{cases}$$



   (a) $\boxed{\text{Circle centered at } (0,0) \text{, traced clockwise.}}$

   (b) Circle centered at $(0,0)$, traced counterclockwise.

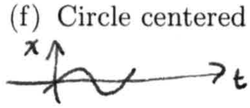
   (c) Circle centered at $(0,1)$, traced clockwise.

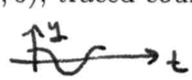
   (d) Circle centered at $(0,1)$, traced counterclockwise.

   (e) Circle centered at $(1,0)$, traced clockwise.

   (f) Circle centered at $(1,0)$, traced counterclockwise.

t	x	y
0	0	1
π/2	1	0
π	0	-1
3π/2	-1	0
2π	0	1





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6. Find the curve described by parametric equations

$$\begin{cases} x(t) = \sqrt{t+1} & 0 \le t \\ y(t) = e^{2t} \end{cases}$$

   (a) $y = \sqrt{e^{2x} + 1}$

   (b) $\boxed{y = e^{2x^2 - 2}}$

   (c) $y = \sqrt{\frac{\ln(x)}{2} + 1}$

   (d) $y = e^{2 - 2\sqrt{x}}$

   (e) $y = e^{\sqrt{2y+2}}$

$$x = \sqrt{t+1} \Rightarrow x^2 = t+1 \Rightarrow x^2 - 1 = t$$

$$y = e^{2t} \Rightarrow y = e^{2(x^2-1)} = e^{2x^2-2}$$

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7. (2 points) Find the Maclaurin series for $f(x) = \int e^{\frac{-x^2}{2}}\, dx$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{\left(\frac{-x^2}{2}\right)} = \sum_{n=0}^{\infty} \frac{\left(\frac{-x^2}{2}\right)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n}}{2^n \cdot n!}$$

$$\int e^{\frac{-x^2}{2}}\, dx = \int \left(\sum_{n=0}^{\infty} \frac{(-1)^n\, x^{2n}}{2^n \cdot n!} \right) dx$$

$$= C + \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n \cdot n!} \left(\int x^{2n}\, dx \right)$$

$$\boxed{= C + \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n \cdot n!} \, \frac{x^{2n+1}}{(2n+1)}}$$

8. (2 points) Find the 3rd degree Taylor Polynomial for $f(x) = x^{-2}$ centered around $x = 2$.

n	$f^{(n)}(x)$	$f^{(n)}(2)$
0	x^{-2}	$\frac{1}{4}$
1	$(-2)\, x^{-3}$	$\frac{-2}{8}$
2	$(-2)(-3)\, x^{-4}$	$\frac{6}{16}$
3	$(-2)(-3)(-4)\, x^{-5}$	$\frac{-24}{32}$

$$T_3(x) = f(2) + \frac{f'(2)}{1!}(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3$$

$$= \frac{1}{4} + \frac{\frac{-2}{8}}{1!}(x-2) + \frac{\frac{6}{16}}{2!}(x-2)^2 + \frac{\frac{-24}{32}}{3!}(x-2)^3$$

$$\boxed{T_3(x) = \frac{1}{4} - \frac{2}{8}(x-2) + \frac{3}{16}(x-2)^2 - \frac{4}{32}(x-2)^3}$$