Math 141 Quiz #3 November 15, 2012

Name: Key Section: ____________

There are 6 multiple choice problems, and 2 short answer problems. **Please mark all multiple choice answers in the box provided.** You have 30 minutes to complete the quiz.

Question	Answer
1	C
2	E
3	A
4	B
5	E
6	A

1. Let $f(x) = \sqrt{x^2+x+2}$. Compute $(f^{-1})'(2)$.

   (a) $\frac{7}{9}$

   (b) 1

   (c) $\frac{4}{3}$

   (d) 0

   (e) $\frac{2}{5}$

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{\frac{1}{2}\cdot\frac{1}{2}\cdot 3} = \frac{4}{3}$$

$$f^{-1}(2) = x \Leftrightarrow 2 = f(x) = \sqrt{x^2+x+2} \Leftrightarrow 4 = x^2+x+2 \Leftrightarrow x = 1$$

$$f'(x) = \frac{1}{2}\cdot\frac{1}{\sqrt{x^2+x+2}}\cdot(2x+1)$$

2. Differentiate $f(x) = e^{x\cdot\sec(x)}$.

   (a) $x\cdot\tan^2(x)\cdot e^{x\cdot\sec(x)} + \sec(x)\cdot e^{x\cdot\sec(x)}$

   (b) $x\cdot\sec^2(x)\cdot e^{x\cdot\sec(x)}$

   (c) $\sec(x)\cdot e^{x\cdot\sec(x)} + \sec(x)\cdot\tan(x)\cdot e^{x\cdot\sec(x)}$

   (d) $x\cdot\sec(x)\cdot\tan(x)\cdot e^{x\cdot\sec(x)}$

   (e) $\sec(x)\cdot e^{x\cdot\sec(x)} + x\cdot\sec(x)\cdot\tan(x)\cdot e^{x\cdot\sec(x)}$

$$f'(x) = e^{x\cdot\sec(x)}\left(\sec(x) + x\cdot\sec(x)\tan(x)\right)$$
$$= \sec(x)\,e^{x\cdot\sec(x)} + x\cdot\sec(x)\tan(x)\cdot e^{x\cdot\sec(x)}$$

3. Find the limit $\lim_{x\to 0} e^{-1/x^2}$

   (a) 0

   (b) 1

   (c) 2

   (d) e

   (e) Diverges

as $x \to 0$

$\frac{1}{x^2} \to \infty$

$\frac{-1}{x^2} \to -\infty$.

as $\frac{-1}{x^2} \to -\infty$

$\frac{1}{e^{1/x^2}} = e^{-1/x^2} \to 0$

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4. Find the sum of the series $\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\frac{16}{81}+\dots$ $= \frac{a}{1-r} = \frac{\frac{2}{3}}{1-\frac{2}{3}} = \frac{\frac{2}{3}}{\frac{1}{3}} = 2$

   (a) 0
   (b) 2
   (c) $\frac{7}{9}$
   (d) $\frac{3}{4}$
   (e) Diverges.

geometric with first term $a=\frac{2}{3}$

and common ratio $\frac{2}{3}=r$

$|r|<1 \Rightarrow$ the series converges & equals $\frac{a}{1-r}=2$

5. Find the limit $\lim_{x\to\infty}\left[\ln(x^2+1)-\ln(x+2)\right]$.

   (a) 0
   (b) $\ln(\frac{1}{2})$
   (c) $\frac{1}{2}$
   (d) $\ln(2)$
   (e) $\boxed{\text{Diverges}}$

$=\lim_{x\to\infty} \ln\left[\frac{x^2+1}{x+2}\right].$

$\boxed{\text{as } x\to\infty,\ \lim_{x\to\infty}\frac{x^2+1}{x+2}=\infty \quad \& \text{ as } u\to\infty\ \lim_{u\to\infty}\ln(u)=\infty}$

$=\infty$

6. Determine if the sequence converges or diverges. If it converges, find its limit.

$$\left\{\frac{2^n+(-1)^n}{3^n}\right\}_{n=1}^{\infty}$$

   (a) $\boxed{0}$
   (b) $\frac{1}{2}$
   (c) 1
   (d) $\frac{4}{7}$
   (e) Diverges

$a_n=\frac{2^n+(-1)^n}{3^n} = \frac{2^n}{3^n}\cdot\frac{1+\frac{(-1)^n}{2^n}}{1} = \left(\frac{2}{3}\right)^n\cdot\frac{1+\left(\frac{-1}{2}\right)^n}{1}$

$\lim_{n\to\infty} a_n = 0$

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7. (2 points). Let $f(x) = \frac{x}{9+x^2}$.

(1) Represent $f(x)$ as a power series, and

(2) give its radius of convergence.

$$\frac{x}{9+x^2} = \frac{x}{9}\cdot\frac{1}{1+\frac{x^2}{9}} = \frac{x}{9}\cdot\frac{1}{1-\left(-\frac{x^2}{9}\right)}$$

$$= \frac{x}{9}\cdot\sum_{n=0}^{\infty}\left(\frac{-x^2}{9}\right)^n$$

(constant WRT the sum)

$$\boxed{= \sum_{n=0}^{\infty}\frac{(-1)^n\, x^{2n+1}}{9^{n+1}}}$$

converges for $\left|\frac{-x^2}{9}\right| < 1$

$\Downarrow$

converges for $\left|\frac{-x^2}{9}\right| < 1$

for $\left|\frac{x}{3}\right|^2 < 1$

for $-3 < x < 3$

$\Rightarrow$ Radius of convergence $= 3$

8. (2 points). Find the interval of convergence of the following power series. Be sure to determine what happens at the endpoints.

$$\sum_{n=1}^{\infty}\frac{x^n}{n\cdot 2^n}$$

Ratio Test:

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{(n+1)\,2^{n+1}}\cdot\frac{n\cdot 2^n}{x^n}\right| = \left|\frac{x}{2}\cdot\frac{n}{(n+1)}\right|$$

the series converges

$$\Leftrightarrow \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x}{2}\right| < 1$$

$\Leftrightarrow$

$|x| < 2$

$\Leftrightarrow$

$-2 < x < 2$

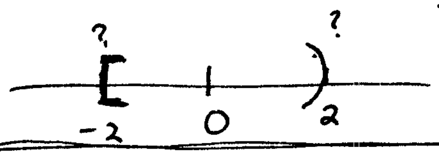
check endpoints

$x = 2 \Rightarrow \sum_{n=0}^{\infty}\frac{2^n}{n\cdot 2^n} = \sum_{n=1}^{\infty}\frac{1}{n}$

diverges, p-series $p=1$

$x = -2 \Rightarrow \sum_{n=0}^{\infty}\frac{(-2)^n}{n\cdot 2^n} = \sum_{n=0}^{\infty}\frac{(-1)^n}{n}$

converges, alternating series test



Interval of Convergence $= [-2, 2)$