Quiz #2 Math 141 Friday, March 1

Name: Key Section: ______________

Question	Answer
1	B
2	A
3	D
4	B
5	F
6	A

There are 6 multiple choice problems, and 2 short answer problems.

Mark **multiple choice answers in the box**.

Show all work for short answer questions.

1. Find the limit of the *sequence* with n^{th} term

$$a_n = n \cdot \tan\left(\frac{1}{n}\right)$$

(a) 0

$\boxed{\text{(b) } 1}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2}$

(e) Diverges

<u>Notice:</u> • $\lim\limits_{x\to\infty} \underset{\infty}{x} \cdot \boxed{\underset{0}{\tan\left(\frac{1}{x}\right)}} = \lim\limits_{x\to\infty} \left(\frac{\tan\left(\frac{1}{x}\right) \to 0}{\frac{1}{x} \to 0}\right)$

$\overset{H}{=} \lim\limits_{x\to\infty} \frac{\sec^2\left(\frac{1}{x}\right)\cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = \lim\limits_{x\to\infty} \boxed{\sec^2\left(\frac{1}{x}\right)} \to 1$

$= 1$

Because $\boxed{a_n = n\cdot\tan\left(\frac{1}{n}\right)}$ and because $\boxed{\text{the function limit converges}}$

The Sequence converges to 1

2. Find the limit of the *sequence* with n^{th} term

$$a_n = \sin(n\cdot\pi)$$

$\boxed{\text{(a) } 0}$

(b) 1

(c) $\frac{\sqrt{2}}{2}$

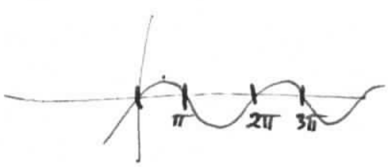
(d) $\frac{3\pi}{2}$

(e) Diverges

<u>Notice:</u> $\{a_1, a_2, a_3, \dots\}$

$= \{\sin(\pi), \sin(2\pi), \sin(3\pi), \dots\}$

$= \{0, 0, 0, \dots\}$



$a_n = 0$ for all n

So the sequence converges to 0.

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3. Find the following *function* limit

$$\lim_{x\to\infty}(1+x^2)^{\frac{1}{\ln(x)}}$$

(a) 2

(b) $e^{\frac{1}{2}}$

(c) $\frac{1}{2}$

$\boxed{\text{(d) } e^2}$

(e) Diverges

let $y = (1+x^2)^{\frac{1}{\ln(x)}}$

$$\lim_{x\to\infty}\ln(y) = \lim_{x\to\infty}\frac{1}{\ln(x)}\cdot\ln\left(1+x^2\right)$$

$$= \lim_{x\to\infty}\frac{\ln(1+x^2)}{\ln(x)}\begin{matrix}\to\infty\\ \to\infty\end{matrix} \overset{H}{=} \lim_{x\to\infty}\frac{\frac{1}{1+x^2}\cdot 2x}{\frac{1}{x}}$$

$$= \lim_{x\to\infty}\frac{2x^2}{1+x^2} = \ldots = 2 = \lim_{x\to\infty}\ln(y)$$

So $\lim_{x\to\infty} y = e^2$

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4. Does the *series* $\sum_{n=1}^{\infty} 2^{n+1}\cdot 3^{-2n}$ converge? If so, what does it converge to?

(a) 2

$\boxed{\text{(b) } \frac{4}{7}}$

(c) $\frac{1}{4}$

(d) $\frac{5}{7}$

(e) Diverges

$$= \sum_{n=1}^{\infty} 2^2\cdot 2^{n-1}\cdot\frac{1}{(3^2)^n}$$

$$= \sum_{n=1}^{\infty}\left(\frac{2^2\cdot 2^{n-1}}{3^2\cdot(3^2)^{n-1}}\right) = \sum_{n=1}^{\infty}\underbrace{\frac{2^2}{3^2}}_{a}\left(\underbrace{\frac{2}{3^2}}_{r}\right)^{n-1}$$

$|r| < 1 \Rightarrow$ the sum $= \frac{a}{1-r}$

$$= \frac{\frac{4}{9}}{1-\frac{2}{9}} = \frac{\frac{4}{9}}{\frac{7}{9}} = \frac{4}{7}$$

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5. Does the *series* $\sum_{n=1}^{\infty} \ln\left(\tan^{-1}(n)\right)$ converge? Why?

(a) Converges - the telescoping series test.

(b) Diverges - the telescoping series test.

(c) Converges - the geometric series test.

(d) Diverges - the geometric series test.

(e) Converges - the divergence test.

$\boxed{\text{(f) Diverges - the divergence test.}}$

Notice: as $n \to \infty$

$\tan^{-1}(n) \to \frac{\pi}{2}$

so $\ln(\tan^{-1}(n)) \to \boxed{\ln\left(\frac{\pi}{2}\right) \neq 0}$

this is by the divergence test

Because the n^{th} term ($= \ln(\tan^{-1}(n))$) doesn't go to 0 the $\boxed{\text{series}}$ ($= \sum_{n=1}^{\infty} \ln(\tan^{-1}(n))$) must diverge

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6. Compute the following *function* limit

$\boxed{\text{(a) } \tfrac{1}{2}}$

(b) 1

(c) $-\frac{1}{2}$

(d) 0

(e) Diverges

$$\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$$

$\frac{1}{x} \to \infty$, $\frac{1}{e^x-1} \to \infty$

indeterminate of type $\infty - \infty$

$$= \lim_{x \to 0^+} \frac{(e^x - 1) - x}{x(e^x - 1)} \quad \begin{matrix} \to 0 \\ \to 0 \end{matrix}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{e^x - 1}{\underbrace{e^x - 1}_{\to 0} + \underbrace{x \cdot e^x}_{\to 0}} \quad \begin{matrix} \to 0 \\ \to 0 \end{matrix}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{e^x}{e^x + e^x + x \cdot e^x} = \frac{1}{2}$$

$e^x \to 1$, $e^x \to 1$, $e^x \to 1$, $x \cdot e^x \to 0$

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7. (2 points) Does the *series* $\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$ converge? If so, what does it converge to?

Remember $\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)} = \lim_{n\to\infty} S_n$ ← sum of first n terms $= \sum_{i=1}^{n} \frac{1}{(i+2)(i+3)}$

[redacted]

Rewrite i^{th} term so later terms cancel earlier terms

$$\frac{1}{(i+2)(i+3)} = \frac{A}{i+2} + \frac{B}{i+3} = \frac{1}{i+2} - \frac{1}{i+3}$$

$$1 = A(i+3) + B(i+2)$$

$i=-2 \Rightarrow A=1$

$i=-3 \Rightarrow B=-1$

write out and simplify S_n

$$S_n = \left[\begin{array}{ll} \cancel{\tfrac{1}{3}} - \cancel{\tfrac{1}{4}} & \leftarrow i=1 \\ + \cancel{\tfrac{1}{4}} - \cancel{\tfrac{1}{5}} & \leftarrow i=2 \\ + \cancel{\tfrac{1}{5}} - \cancel{\tfrac{1}{6}} & \leftarrow i=3 \\ \vdots & \\ + \cancel{\tfrac{1}{n+1}} - \cancel{\tfrac{1}{n+2}} & \leftarrow i=n-1 \\ \cancel{\tfrac{1}{n+2}} - \tfrac{1}{n+3} & \leftarrow i=n \end{array}\right] = \underbrace{\frac{1}{3} - \frac{1}{n+3}}_{\text{this is the sum of the first n terms}}$$

$$\sum_{n=1}^{\infty} = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[\frac{1}{3} - \frac{1}{n+3}\right] = \frac{1}{3}$$

↑ sum of first n terms

$\boxed{\text{the series converges to } \frac{1}{3}}$

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8. (2 points) Does the *integral* $\int_1^\infty \frac{1}{x^{.5}(x+2)}\,dx$ converge?

Compare

$$\frac{1}{x^{.5}(x+2)} \leq \frac{1}{x^{.5}\cdot x} = \frac{1}{x^{1.5}}$$

Compute: comparison integral

$$\int_1^\infty \frac{1}{x^{1.5}}\,dx = \lim_{t\to\infty} \int_1^t x^{-1.5}\,dx$$

negative

$$= \lim_{t\to\infty} \left[\frac{x^{-.5}}{-.5} \right]_1^t$$

the comparison integral converges

a larger integral converges

so $\boxed{\text{original integral converges}}$

(by the integral comparison test)