Quiz #2 Review

Math 141: Section 33.

February 23, 2013

1. **Indeterminate Forms/L'Hopital's Rule**
   (a) Remember that you should **not** try to do arithmetic with ∞. When you are given a limit $\lim_{x\to a} f(x)$, you should ask yourself "as x goes to a, where does $f(x)$ go?"
   (b) Remember the definition of an "indeterminate form."
   Indeterminate forms include quotients ($\frac{0}{0}$ and $\frac{\infty}{\infty}$), products ($0\cdot\infty$), differences ($\infty-\infty$), and exponents (1^{∞}, 0^{∞}, ∞^0, and 0^0).
   Do not attempt to do arithmetic with indeterminate forms!!!
   (c) Be able to apply L'Hopital's rule when appropriate.
      i. L'Hopital's rule applies to indeterminate forms of type $\frac{0}{0}$ and $\frac{\infty}{\infty}$.
      ii. First turn indeterminate products and differences into a fraction of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$.
      iii. Evaluate indeterminate powers using the **three step procedure**.
      (1) Rewrite $f^g = e^{g\cdot\ln(f)}$, (2) compute $\lim[g\cdot\ln(f)] = A$, and (3) the answer is e^A.
   (d) Remember that there are alternatives to L'Hopital's rule which are often faster or better (simplifying, multiply by a fraction such as $\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$, etc).
2. **Relative Rates of Growth**
   (a) Know the definition of "$f(x)$ grows faster than $g(x)$," which is written $f \gg g$.
   (b) Know the definition of "$f(x)$ grows at the same rate as $g(x)$," which is written $f \asymp g$.
   (c) Know relative rates of growth of certain functions:
      i. $\ln(\ln(x))^a \ll \ln(x)^b$ for any $a, b > 0$.
      ii. $\ln(x)^a \ll x^b$ for any $a, b > 0$
      iii. $x^a \ll x^b$ for any $b > a > 0$.
      iv. $a^x \ll b^x$ for any $b > a > 1$
      v. $a^x \ll x^x$ for any $a > 1$.
3. **Improper Integrals**
   (a) Know the *definition* of when an improper integral converges, and when it diverges.
   (b) Know the 3 types of "infinite" integrals.
   (c) Be able to turn each type of "infinite integral" into the limit of proper integrals.
   (d) Know the 3 types of "discontinuous integrand" integrals.

(e) Be able to turn each type of integral with a "discontinuous integrand" into the limit of proper integrals. (Be careful to approach the limit from the correct direction.)

(f) Remember to *first* evaluate the anti-deriviative, and *then* to take the limit.

(g) Be able to use the comparison test (for integrals).

i. Be able to make straightforward comparisons, such as making a fraction bigger/smaller by simplifying the top/bottom.

ii. You must use a *smaller* function that *diverges* to show *divergence*, and

iii. You must use a *larger* function that *converges* to show *convergence*.

4. **Sequences**

(a) **Double check that you are looking at a *sequence*** (and not at a series).

(b) Be able to use a formula for a_n to write first few terms of a *sequence*.

(c) Know that "$\lim_{n\to\infty} a_n = L$" means "the numbers a_n converge to L as n goes to ∞."

(d) Know the r such that the *sequence* $\{r^n\}_{n=1}^{\infty}$ converges. Know what it converges to.

(e) Know the methods for computing limits of *sequences*.

i. If $f(x)$ is a function and $a_n = f(n)$,
then as long as $\lim_{x\to\infty} f(x) = L$ is finite, then the *sequence* $\{a_n\}$ converges to L.

ii. If the *sequence* $\{a_n\}$ converges to the finite number L and if f is a function that is continuous at L, then $\lim_{n\to\infty} f(a_n) = f(\lim_{n\to\infty} a_n) = f(L)$.

iii. Know the "limit laws for *sequences*" on page 717 of the textbook.

(f) Be able to use the "squeeze" theorem to show that a *sequence* converges (to some #)

(g) Be able to use a comparison with another *sequence* to show that a *sequence* diverges.

5. **Series**

(a) **Double check that you are looking at a *series*** (and not at a sequence).

(b) Know that a *series* is the sum of an ordered list of numbers.

(c) Know that a *series* is the limit of its partial sums $s_n = a_1 + a_2 + \cdots + a_n = \sum_{i=1}^{n} a_i$.

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \sum_{i=1}^{n} a_i$$

(d) Know the definition of "geometric series," of the "first term," and of the "common ratio."

(e) Be able to find the first term and common ratio of complicated looking geometric series.

(f) Be able to tell if a geometric series diverges (if $|r| \geq 1$) or converges (if $|r| < 1$).
If it converges, be able to compute the value that the *series* converges to.

(g) Be able to compute the value of a telescoping *series*.

(h) Be able to use the "divergence test". For example, be able to show that the *series* $\sum_{n=1}^{\infty} \frac{x^4+1}{x^4+3x^3+2}$ diverges.

(i) Know that the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.