Math 141 Name: Key Quiz #2 Thursday, October 18, 2012

Section: ____________

There are 6 multiple choice problems, and 3 short answer problems. **Please mark all multiple choice answers in the box provided.**

| Question | Answer |
|---|---|
| 1 | E |
| 2 | A |
| 3 | C |
| 4 | E |
| 5 | E |
| 6 | A |

1. $\int_{-\infty}^{\infty} \frac{1}{2}x\,dx$ $= \int_0^\infty \frac{x}{2}\,dx + \int_{-\infty}^{0} \frac{x}{2}\,dx$

(a) 1

(b) $\frac{1}{2}$

(c) 0

(d) $\frac{3}{2}$

$\boxed{\text{(e)}}$ The integral diverges.

$= \lim_{t\to\infty} \int_0^t \frac{x}{2}\,dx + \cdots$

$= \underbrace{\lim_{t\to\infty} \left[t^2 - 0^2\right]}_{\text{diverges}} + \cdots$

$\Rightarrow$ the original integral diverges.

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2. Does $\lim_{x\to 0+} x^x$ converge? If so, what does it converge to?

$\boxed{\text{(a)}}$ 1

(b) $e^{-1}$

(c) 0

(d) $e$

(e) The limit diverges.

$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{\boxed{x\cdot\ln(x)}} = e^{\boxed{0}} = 1$

$\lim_{x\to 0^+} x\cdot \ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{\frac{1}{x}}$ $\xrightarrow{} \frac{-\infty}{\infty}$

$\stackrel{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} \cdot \frac{x^2}{x^2} = \lim_{x\to 0^+} \frac{x}{-1} = \boxed{0}$

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3. Let $a_n = n\cdot\sin\left(\frac{1}{n}\right)$. Find $\lim_{n\to\infty} a_n$.

$= \lim_{n\to\infty} n\cdot \sin\left(\frac{1}{n}\right)$ $\to \infty \cdot 0$

(a) 0

(b) $\frac{\pi}{2}$

$\boxed{\text{(c)}}$ 1

(d) $-\frac{\pi}{2}$

(e) The limit diverges.

$= \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}$ $\to \frac{0}{0}$ $\stackrel{H}{=} \lim_{n\to\infty} \frac{\cos\left(\frac{1}{n}\right)\cdot\frac{-1}{n^2}}{\frac{-1}{n^2}} \left(\frac{-n^2}{-n^2}\right)$

$= \lim_{n\to\infty} \cos\left(\frac{1}{n}\right)$

$= \cos(0) = 1$

Name: ______________________ Section: ______________________

<u>Note:</u> this series is geometric.

4. $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{3\cdot 4^{n-1}}{3^n} = \frac{3}{3} + \frac{-3\cdot 4}{3^2} + \frac{3\cdot 4^2}{3^3} + \cdots$

   (a) $\frac{7}{9}$

   (b) $\frac{3}{7}$

   (c) $\frac{7}{3}$

   (d) $\frac{9}{7}$

   $\boxed{\text{(e)}}$ The series diverges.

$a = 1$

$r = \dfrac{\frac{-3\cdot 4}{3^2}}{\frac{3}{3}} = \dfrac{-4}{3}$

$|r| = \frac{4}{3} > 1$

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5. $\sum_{n=1}^{\infty} \ln\left(\frac{n+2}{n+1}\right)$ [redacted] $= \lim_{n\to\infty} \boxed{S_n}$

   (a) $-\ln(2)$

   (b) $0$

   (c) $1$

   (d) $\ln(3) - \ln(2)$

   $\boxed{\text{(e)}}$ The series diverges.

$\boxed{S_n} = \sum_{i=1}^{n} \ln\left(\frac{i+2}{i+1}\right) = \sum_{i=1}^{n} \ln(i+2) - \ln(i+1)$

$= \ln(\cancel{3}) - \ln(2) \leftarrow i=1$

$+ \ln(\cancel{4}) - \ln(\cancel{3}) \leftarrow i=2$

$+ \ln(\cancel{5}) - \ln(\cancel{4}) \leftarrow i=3$

$\vdots$

$+ \ln(\cancel{n+1}) - \ln(\cancel{n}) \leftarrow i=n-1$

$+ \ln(n+2) - \ln(\cancel{n+1}) \leftarrow i=n$

$S_n = \ln(n+2) - \ln(2)$

$\lim_{n\to\infty} S_n = \infty = \sum_{n=1}^{\infty} a_n$

<u>Note:</u> this series is geometric.

6. $\sum_{n=1}^{\infty} 5\,\frac{2^{n-1}}{3^{2n}} = \frac{5}{3^2} + \frac{5\cdot 2}{3^4} + \frac{5\cdot 2^2}{3^6} + \cdots$

   $\boxed{\text{(a)}}$ $\frac{5}{7}$

   (b) $\frac{35}{9}$

   (c) $\frac{9}{7}$

   (d) $\frac{45}{7}$

   (e) The series diverges.

$a = \frac{5}{9}$ . [redacted]

$r = \frac{5\cdot 2}{3^4}\cdot\frac{3^2}{5} = \frac{2}{3^2} = \frac{2}{9}$

$|r| = \frac{2}{9} < 1 \Rightarrow \sum_{n=1}^{\infty} 5\,\frac{2^{n-1}}{3^{2n}} = \dfrac{\frac{5}{9}}{1-\frac{2}{9}} = \dfrac{\frac{5}{9}}{\frac{7}{9}} = \frac{5}{7}$

Name: ______________________ Section: ______________________

7. (2 points) Use the comparison test to determine if the following integral converges. You must show all work to get full credit.

$$\int_1^\infty \frac{x}{\sqrt{x^4 - x}}\, dx$$

$$\frac{x}{\sqrt{x^4-x}} \geq \frac{x}{\sqrt{x^4}} = \frac{x}{x^2} = \frac{1}{x}$$

and

$$\int_1^\infty \frac{1}{x}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{x}\,dx = \lim_{t\to\infty} \left(\ln|t| - \ln|1|\right)$$

$$= \infty \quad \text{(Diverges)}$$

the comparison integral is <u>smaller</u> and <u>diverges</u>

$\Rightarrow$ the original integral <u>diverges</u>

8. (2 points) First, write out the first three terms of the sequence $\{(-1)^{n-1} + \cos(n\pi)\}$ and **simplify each term** completely. Then, compute $\lim\limits_{n\to\infty} a_n$.

$$\left\{(-1)^0 + \cos(\pi),\ (-1)^1 + \cos(2\pi),\ (-1)^2 + \cos(3\pi),\ \dots\right\}$$

$$= \left\{1 + (-1),\ (-1) + 1,\ 1 + (-1),\ \dots\right\}$$

$$= \left\{0,\ 0,\ 0,\ \dots\right\}$$

so $\lim\limits_{n\to\infty} a_n = 0$