Quiz #1   Math 141   January 25, 2013

Name: Key   Section: ____________

Question	Answer
1	C
2	C
3	C
4	E
5	A

There are 5 multiple choice problems, and 2 short answer problems. You have 25 minutes to complete the quiz.

Please mark all multiple choice answers in the box provided.

1. Completely evaluate the following definite integral:

$$\int_0^1 (e^{2x}+1)\,dx = \int_0^1 e^{2x}\,dx + \int_0^1 1\cdot dx$$

$$= \left[\frac{e^{2x}}{2} + x\right]_0^1$$

$$= \left(\frac{e^2}{2}+1\right) - \left(\frac{e^0}{2}+0\right) = \frac{e^2}{2} + \frac{1}{2}$$

(a) $e^2 - \frac{1}{2}$

(b) $2 \cdot e^2 - 1$

(c) $\boxed{\frac{e^2}{2} + \frac{1}{2}}$

(d) $2 \cdot e^2 + 2$

(e) $\frac{e^2}{2} + 3$

2. If $f(x) = x^{\tan(x)}$, find an equation for $f'(x)$.

(a) $x^{\tan(x)} \cdot \sec^2(x) \cdot \ln(x) + \frac{\tan(x)}{x}$

(b) $x^{\tan(x)} \left(\sec^2(x) \cdot \ln(x) + \tan(x) \cdot \ln(x)\right)$

(c) $\boxed{x^{\tan(x)} \left(\sec^2(x) \cdot \ln(x) + \frac{\tan(x)}{x}\right)}$

(d) $x^{\tan(x)} \cdot \frac{\sec^2(x) + \ln(x)}{x}$

(e) $x^{\tan(x)} \left(\frac{\sec^2(x)}{\ln(x)} + \tan(x) \cdot \ln(x)\right)$

$$y = x^{\tan(x)}$$

$$\ln(y) = \ln\left(x^{\tan(x)}\right) = \tan(x) \cdot \ln(x)$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \sec^2(x) \cdot \ln(x) + \tan(x) \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = y\left(\sec^2(x) \cdot \ln(x) + \tan(x) \cdot \frac{1}{x}\right)$$

$$y = x^{\tan(x)}$$

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3. Evaluate the following integral:

$$\int \underbrace{\frac{1}{x\sqrt{1-(\ln(x))^2}}\,dx}_{du}$$

(a) $\sec(\ln(x)) + C$

(b) $\sec^{-1}(x) + C$

(c) $\boxed{\sin^{-1}(\ln(x)) + C}$

(d) $\ln(x)\cdot\sin^{-1}(x) + C$

(e) $\sin^{-1}(x) + C$

$\left(\begin{array}{l} u\text{-Sub} \\ u = \ln(x) \\ du = \frac{1}{x}dx \end{array}\right)$

$$= \int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1}(u) + C = \sin^{-1}(\ln(x)) + C$$

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4. Let $f(x) = \log_5(x)\cdot 5^x$. Find $f'(x)$.

(a) $\dfrac{\ln(x)}{\ln(5)}\cdot 5^x + \ln(5)\cdot\log_5(x)\cdot 5^x$

(b) $\ln(5)\cdot\dfrac{5^x}{x} + \log_5(x)\cdot\ln(5)\cdot 5^x$

(c) $\dfrac{5^x}{x} + \log_5(x)\cdot 5^x$

(d) $\dfrac{5^x}{x} + \log_5(x)\cdot 5^x\cdot\ln(5)$

(e) $\boxed{\dfrac{1}{\ln(5)\cdot x}\cdot 5^x + \log_5(x)\cdot 5^x\cdot\ln(5)}$

$$\frac{d}{dx}\left(\log_5(x)\cdot 5^x\right) = \frac{d}{dx}\left(\log_5(x)\right)\cdot 5^x + \log_5(x)\cdot\frac{d}{dx}(5^x)$$

$$= \left(\frac{1}{\ln(5)}\cdot\frac{1}{x}\right)\cdot 5^x + \log_5(x)\cdot\left(5^x\cdot\ln(5)\right)$$

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5. Find the derivative of $y = \ln(e^x\cdot\sin(x))$.

(a) $\boxed{1 + \dfrac{\cos(x)}{\sin(x)}}$

(b) $\dfrac{e^x}{\sin(x)} + \dfrac{\cos(x)}{e^x}$

(c) $\dfrac{\cos(x)}{\sin(x)}$

(d) $\dfrac{1}{\sin(x)} + \dfrac{\cos(x)}{e^x\cdot\sin(x)}$

(e) None of the above.

$$\frac{dy}{dx} = \frac{1}{e^x\cdot\sin(x)}\left(e^x\cdot\sin(x) + e^x\cdot\cos(x)\right)$$

$$= 1 + \frac{\cos(x)}{\sin(x)}$$

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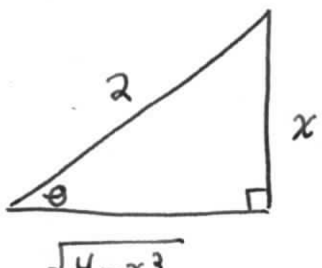
6. (2 points) Suppose that $\sin(\theta) = \frac{x}{2}$ and that $0 < \theta < \frac{\pi}{2}$. Find $\cos(\theta)$.



$$\sin\theta = \frac{opp}{hyp} = \frac{x}{2}$$

$$adj^2 + x^2 = 2^2$$

$$adj^2 = 4 - x^2$$

$$adj = \sqrt{4 - x^2}$$

$$\cos(\theta) = \frac{adj}{hyp} = \frac{\sqrt{4-x^2}}{2}$$

7. Let $f(x) = x^3 + x^2 + x + 1$. Compute $(f^{-1})'(4)$.

$$(f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))}$$

$$f^{-1}(4) = x$$

means: $4 = f(x) = x^3 + x^2 + x + 1$

this is true for $x = 1$

so $4 = f(1)$

so $f^{-1}(4) = 1$

$$f'(x) = 3x^2 + 2x + 1$$

$$f'(f^{-1}(4)) = f'(1) = 3 \cdot 1^2 + 2 \cdot 1 + 1$$

$$f'(1) = 6$$

so $(f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))} = \frac{1}{6}$