Review Sheet #1. Math 141 Parametric and Polar Curves

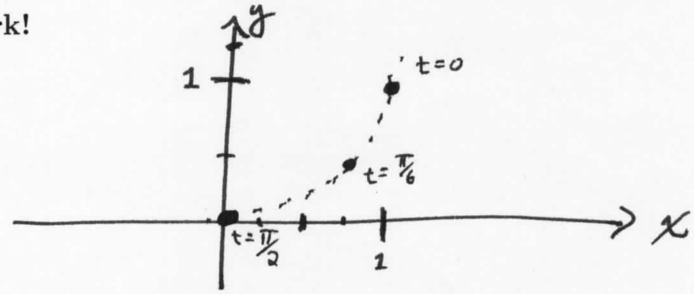
Name: Solutions Section: ________________

1. Consider the **parametric** curve given by the equations

$$\begin{cases} x(t) = \cos^2(t) & 0 \le t \le \frac{\pi}{2} \\ y(t) = 1 - \sin(t) \end{cases}$$

(a) Sketch the curve. **Show your work!**

t	$x(t)$	$y(t)$
0	$1^2 = 1$	$1 - 0 = 1$
$\pi/6$	$\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$	$1 - \frac{1}{2} = \frac{1}{2}$
$\pi/2$	0^2	$1 - 1 = 0$



(b) Find a formula for the slope $\frac{dy}{dx}$ of the curve (for t such that $\frac{dx}{dt} \neq 0$).

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\cos(t)}{2 \cdot \cos(t) \cdot (-\sin(t))} = \frac{1}{2 \cdot \sin(t)}$$

(c) Find the equation for the line tangent the curve at $t = \frac{\pi}{6}$.

slope at $t = \pi/6$: is $\frac{dy}{dx} = \frac{1}{2 \cdot \sin(\pi/6)} = \frac{1}{2 \cdot \frac{1}{2}} = 1$

point of intersection: is $\left(x(\frac{\pi}{6}), y(\frac{\pi}{6})\right) = \left(\frac{3}{4}, \frac{1}{2}\right)$

tangent line is: $y = 1 \cdot \left(x - \frac{3}{4}\right) + \frac{1}{2} = x - \frac{1}{4}$

(d) Find a Cartesian equation for the curve (in terms of just x and y).

Know: $(\sin(t))^2 + (\cos(t))^2 = 1$

Know: $(\cos(t))^2 = x$, and $1 - y = \sin(t)$

so $\boxed{(1-y)^2 + x = 1}$ or, equivalently, $\boxed{(y-1)^2 + x = 1}$

(e) Find a *formula* for the arc-length of the parametric curve. Simplify where possible.

(If you have extra time at the end, try to evaluate the integral).

$$\text{arc-length} = \int_0^{\pi/2} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt = \int_0^{\pi/2} \sqrt{\cos^2(t) + 4\cos^2(t)\sin^2(t)}\, dt$$

Be careful to square ALL [redacted] of $\frac{dx}{dt}$

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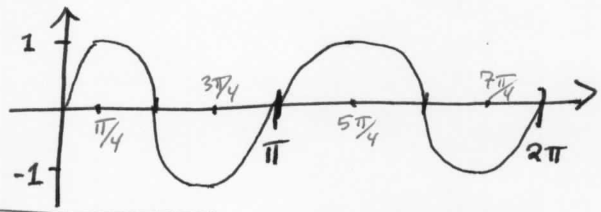
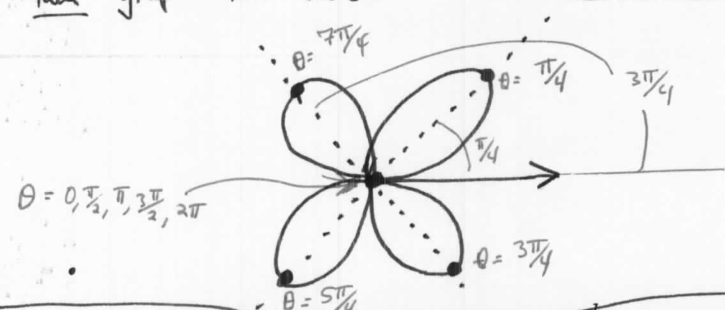
2. Consider the **polar** curve given by the equation $r = \sin(2\theta)$ for $0 \le \theta \le 2\pi$.

(a) Sketch the curve. **Show your work!**

first: graph r vs θ



then graph the curve:



(b) Find a formula for the slope $\dfrac{dy}{dx}$ to the curve (for θ such that $\dfrac{dx}{d\theta} \neq 0$).

$$x(\theta) = r \cdot \cos\theta = (\sin 2\theta) \cdot \cos\theta$$

$$y(\theta) = r \cdot \sin\theta = (\sin(2\theta)) \cdot \sin\theta$$

$$\frac{dx}{d\theta} = 2\cdot\cos(2\theta)\cdot\cos\theta - \sin(2\theta)\cdot\sin\theta$$

$$\frac{dy}{d\theta} = 2\cdot\cos(2\theta)\cdot\sin\theta + \sin(2\theta)\cdot\cos\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cdot\cos(2\theta)\cdot\sin\theta + \sin(2\theta)\cdot\cos\theta}{2\cdot\cos(2\theta)\cdot\cos\theta - \sin(2\theta)\cdot\sin\theta}$$

(c) Find the area enclosed by the curve.

the curve never sweeps out the same area twice $\Rightarrow$ area enclosed $= \displaystyle\int_0^{2\pi} \frac{1}{2} r^2 \, d\theta = \int_0^{2\pi} \frac{1}{2}(\sin(2\theta))^2 \, d\theta$

$$= \int_0^{2\pi} \frac{1}{2}\left(\frac{1-\cos(4\theta)}{4}\right) d\theta = \frac{\theta}{4} - \int_0^{2\pi} \frac{1}{4}\cos(4\theta)\, d\theta$$

$u = 4\theta$

$\Rightarrow du = 4\,d\theta$

$\Rightarrow d\theta = \frac{du}{4}$

$$= \left[\frac{\theta}{4} - \frac{1}{16}\sin(4\theta)\right]_0^{2\pi} = \left(\frac{2\pi}{4} - 0\right) - \left(\frac{1}{16}\cdot 0 - \frac{1}{16}\cdot 0\right)$$

$= \frac{\pi}{2}$ = Area enclosed

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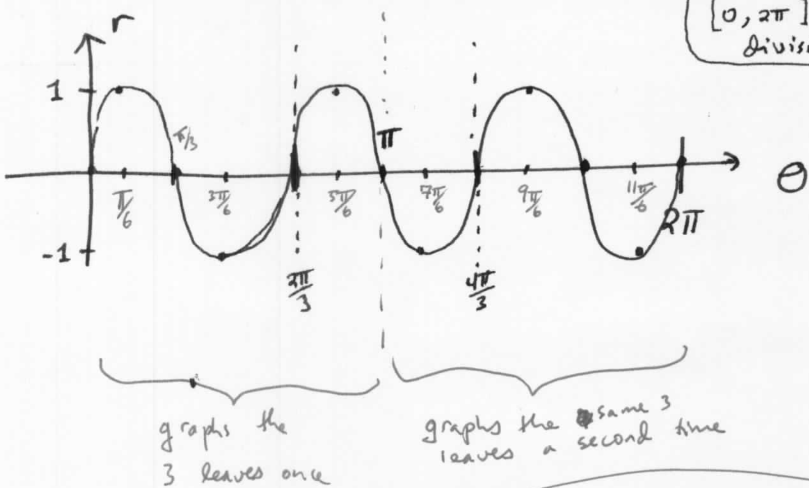
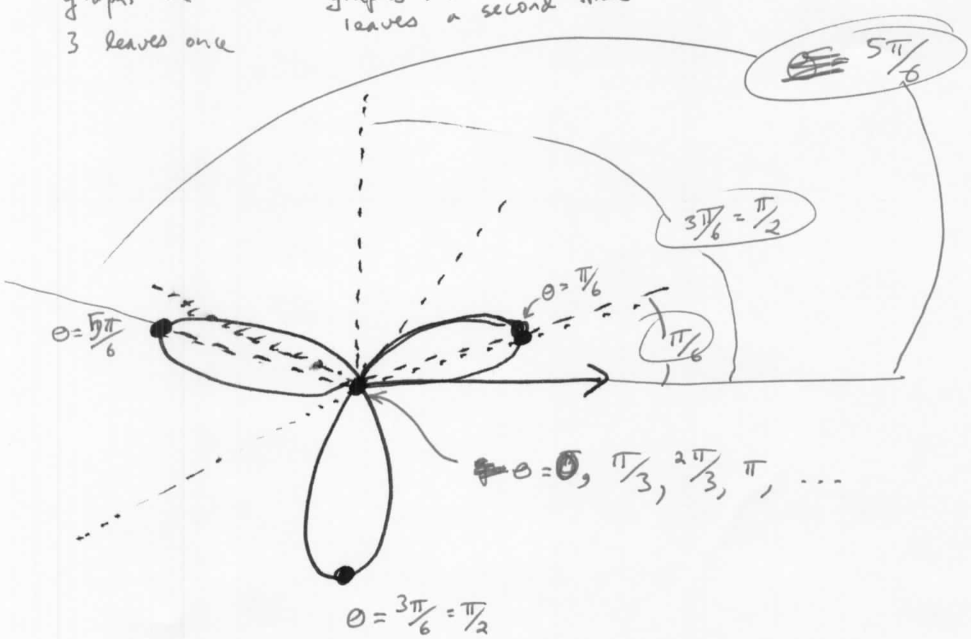
3. Consider the **polar** curve given by the equation $r = \sin(3\theta)$ for $0 \le \theta \le 2\pi$.

   (a) Sketch the curve. **Show your work!**

this will repeat 3 times on $[0, 2\pi]$, so we need 3 major divisions on r vs. θ





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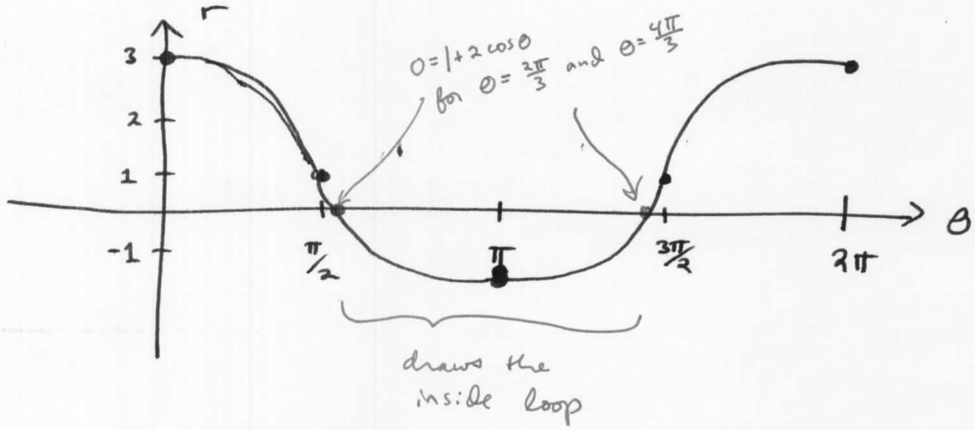
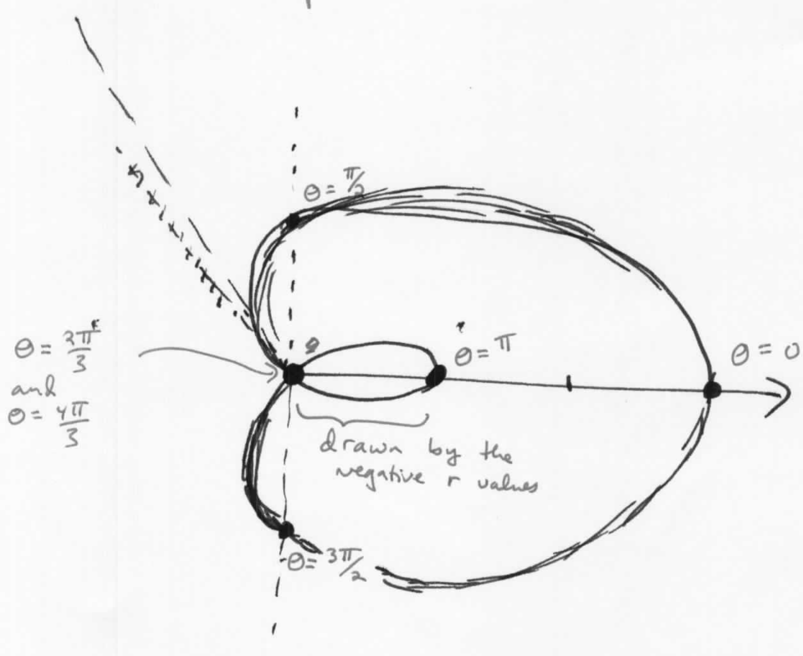
4. Consider the **polar** curve given by the equation $r = 1 + 2\cos(\theta)$ for $0 \leq \theta \leq 2\pi$.

   (a) Sketch the curve. **Show your work!**





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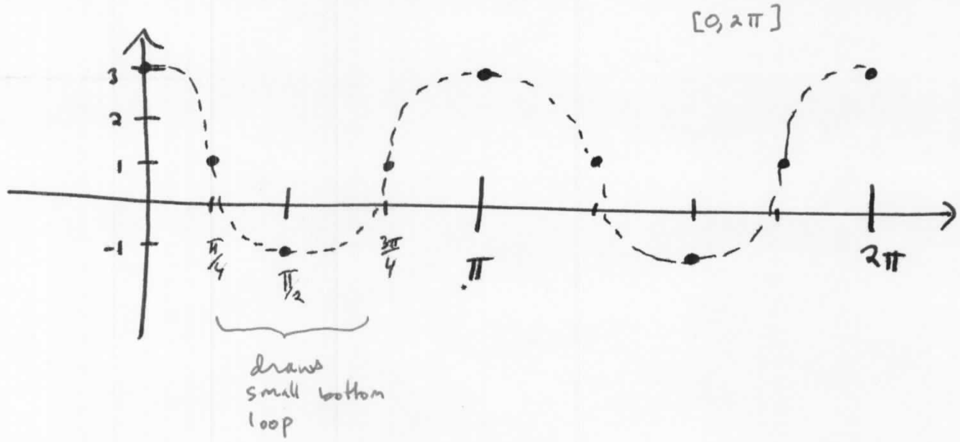
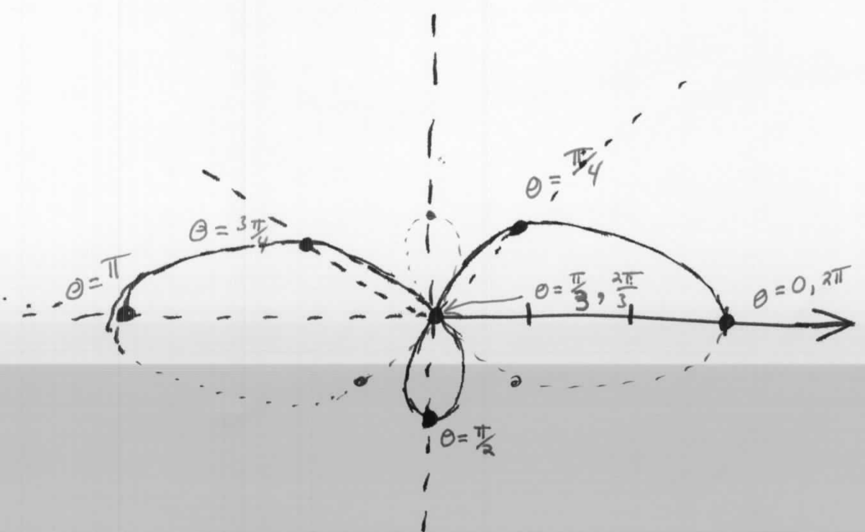
5. Consider the **polar** curve given by the equation $r = 1 + 2\cos(2\theta)$ for $0 \leq \theta \leq 2\pi$.

   (a) Sketch the curve. **Show your work!**

the curve from # 4, repeating twice on $[0, 2\pi]$





continuing, θ between π & 2π draw the mirror image

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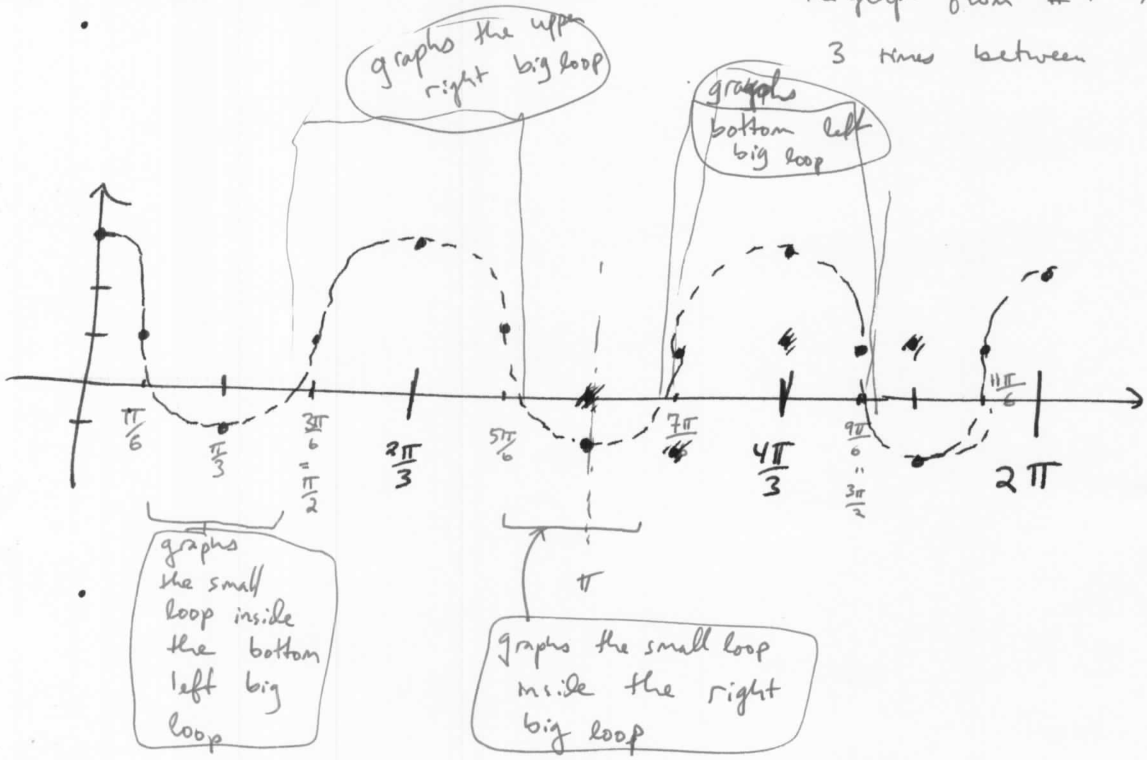
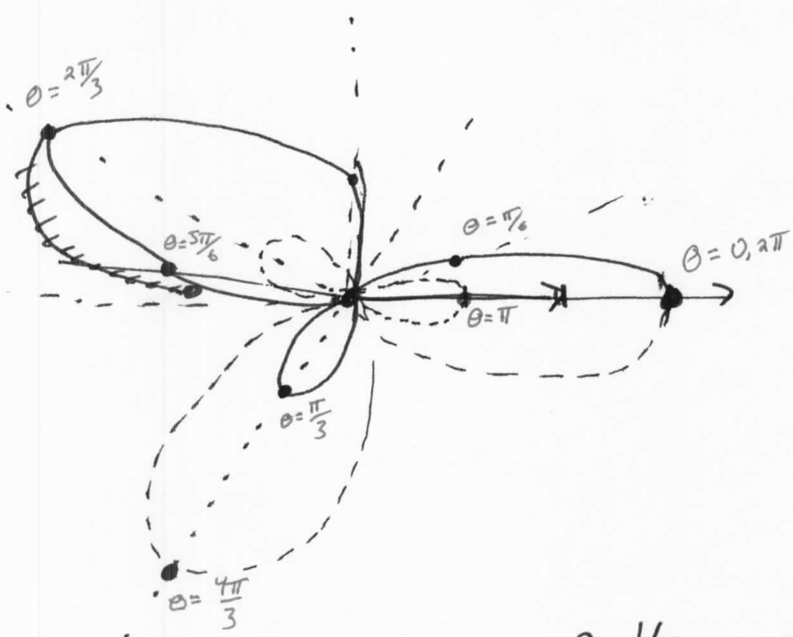
6. Consider the **polar** curve given by the equation $\underbrace{r = 1 + 2\cos(3\theta)}$ for $0 \le \theta \le 2\pi$.

   (a) Sketch the curve. **Show your work!**

   the graph from #4 repeats 3 times between 0 & 2π





Prettier sketch: