Sequence Limits

Notation: $\lim_{n \to \infty} a_n$, limit of $\{a_n\}$.

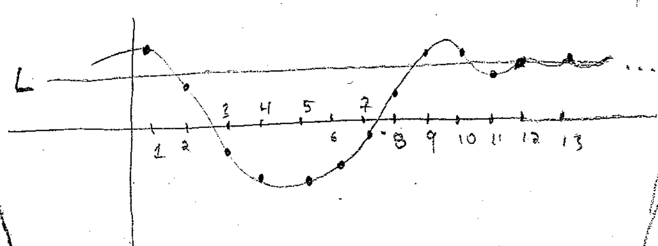
NOTE: n is ALWAYS a counting #

Sequence Limits that are like function limits

If $a_n = f(n)$

and $\lim_{x \to \infty} f(x) = L$

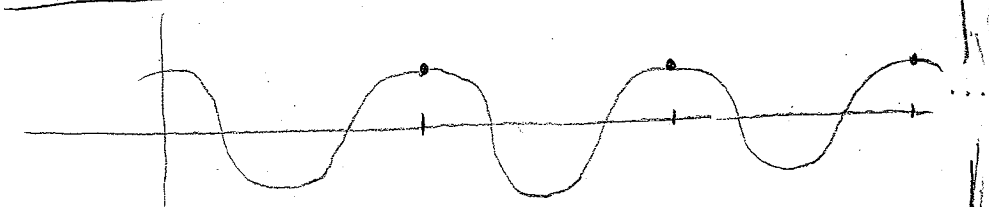
Then $\lim_{n \to \infty} a_n = L$



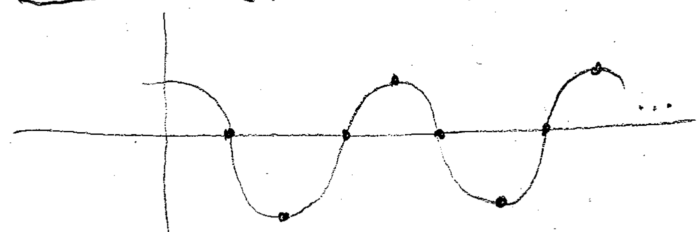
NO overlap!

Sequence limits that are NOT like function limits

Some converge: $a_n = \cos(2\pi n)$



some diverge: $a_n = \cos\left(\frac{\pi n}{2}\right)$



Series

<u>Notation</u>: $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + a_4 + \cdots + a_n + \cdots$

Series Convergence tests

1. Divergence test
2. Special Series
   - geometric
   - p-series
   - alternating
   - telescoping
3. tests for positive series
   - Direct comparison
   - limit comparison
   - Integral test
4. Ratio & Root tests

Absolute convergence/ conditional convergence

KEY:

Involves a sequence limit

Involves a function limit

Basic Series Methods

→ Geometric sums

→ Series arithmetic

→ alternating error estimation

→ Partial Sums

$s_n = a_1 + \cdots + a_n = \sum_{i=1}^{n} a_i$

the <u>whole series</u> is the limit of its partial sums

$\sum_{n=1}^{\infty} a_i = \lim_{n \to \infty} s_n$

(these ideas and methods overlap)

Function Limits

<u>Notation:</u> x, y, z can be any number

Applying function limit tricks to certain sequence limits

Tricky Types of Limits

→ Indeterminate of type $\frac{0}{0}$ and $\frac{\infty}{\infty}$

(⇒ use L'Hopital's Rule or clever rewriting)

→ Indeterminate of type $0 \cdot \infty$, 1^{∞}, ∞^{0}, $\infty - \infty$

(⇒ rewrite until you can use L'Hopital's Rule)

→ Complex combinations of functions

⇒ think carefully.

⇒ use rates of growth to find the fastest term?

⇒ rewrite to be easier to understand?

Improper Integrals

① turn into a limit of proper integrals

② integrate

③ take limit