Eg: What does $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{5^n}$ do?

Important!

$$|a_n| = \left| (-1)^n \frac{n^2}{5^n} \right| = \frac{n^2}{5^n}$$

simplify the ratio

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{\frac{(n+1)^2}{5^{n+1}}}{\frac{n^2}{5^n}} = \frac{(n+1)^2}{5^{n+1}} \cdot \frac{5^n}{n^2}$$

$$= \frac{(n+1)^2}{n^2} \cdot \frac{5^n}{5^{n+1}}$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{(n+1)^2}{n^2} \cdot \frac{1}{5}$$

take the limit

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left(\frac{(n+1)^2}{n^2} \cdot \frac{1}{5} \right)$$

$$\left(\lim_{n\to\infty} \frac{(n+1)^2}{n^2} = \lim_{n\to\infty} \frac{\cancel{n^2}}{\cancel{n^2}} \frac{(1+\frac{1}{n})^2}{1} = 1 \right)$$

$$= (1)\left(\frac{1}{5}\right) < 1$$

$\Rightarrow$ the series converges absolutely by the ratio test

Eg: What does $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$ do?

$$|a_n| = \left|\frac{(-2)^n}{n!}\right| = \frac{2^n}{n!}$$

① simplify the ratio!

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\left(\frac{2^{n+1}}{(n+1)!}\right)}{\left(\frac{2^n}{n!}\right)} = \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}$$

(collect related terms and cancel)

$$= \frac{2^{n+1}}{2^n} \cdot \frac{n!}{(n+1)!}$$

$$= 2 \cdot \frac{\cancel{n} \cdot \cancel{(n-1)} \cdots \cancel{2} \cdot 1}{(n+1) \cdot \cancel{n} \cdot \cancel{(n-1)} \cdots \cancel{2} \cdot 1}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{2}{n+1}$$

② compute the limit

$$\lim_{n \to \infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n \to \infty} \frac{2}{n+1} = 0 < 1$$

$\Rightarrow$ the original series **converges absolutely** by the ratio test