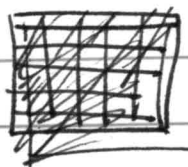
Eg: Does this series converge?

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{2\cdot n^3+n+1}$$

At the beginning of class, we saw as $n\to\infty$ this "behaved" like $\frac{1}{n^{1.5}}$



Limit comparison

$$a_n = \frac{\sqrt{n+2}}{2\cdot n^3+n+1} = \frac{\sqrt{n}\sqrt{1+\frac{2}{n}}}{n^3\left(2+\frac{1}{n}+\frac{1}{n^3}\right)}$$

$$b_n = ? = \frac{\sqrt{n}}{n^3} = \frac{1}{n^{1.5}}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{\sqrt{n}}{n^3}\frac{\sqrt{1+\frac{2}{n}}}{\left(2+\frac{1}{n}+\frac{1}{n^2}\right)}}{\frac{\sqrt{n}}{n^3}} = \lim_{n\to\infty} \frac{\sqrt{1+\frac{2}{n}}}{\left(2+\frac{1}{n}+\frac{1}{n^2}\right)} = \frac{1}{2} \neq 0$$

AND

$$\sum_{n=1}^{\infty} \frac{1}{n^{1.5}} \text{ converges } \left(p\text{-series w/ } p>1\right)$$

$\Rightarrow$ the series converges by limit comparison with $\frac{1}{n^{1.5}}$