Two Ways to Recognize a Geometric Series using the first few terms

Method 1:

Ask ① "what # r Gives $(1^{st}\text{ term})\cdot r = (2^{nd}\text{ term})$?"

and ② "Is $(2^{nd}\text{ term})\cdot r = (3^{rd}\text{ term})$?"

Eg: $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \cdots$

$\boxed{\text{first term } a=5}$

① to get from 5 to $\frac{-10}{3}$

multiply by $\frac{-2}{3}$

② **check:** $\underbrace{\left(\frac{-10}{3}\right)}_{\boxed{2^{nd}\text{ term}}}\cdot\underbrace{\left(\frac{-2}{3}\right)}_{\boxed{r}} = \underbrace{\frac{20}{9}}_{\boxed{3^{rd}\text{ term}}}$ ✓

$$\boxed{\textbf{So: } 5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \cdots = \sum_{n=1}^{\infty} 5\left(\frac{-2}{3}\right)^{n-1}}$$

Method 2: ① **Compute** $r = \dfrac{(2^{nd}\text{ term})}{(1^{st}\text{ term})}$

② **check:** $r = \dfrac{(3^{rd}\text{ term})}{(2^{nd}\text{ term})}$

infinite sum

Three Ways to Describe a Geometric Series using an

Method 1: Write out terms to $\boxed{\text{see}}$ pattern

Eg: $$\sum_{n=1}^{\infty} 2^n 3^{1-n} = 2 + 2\left(2\cdot\tfrac{1}{3}\right) + 2\cdot\left(2\cdot\tfrac{1}{3}\right)\cdot\left(2\cdot\tfrac{1}{3}\right) + \cdots$$

$$= 2 + 2\cdot\left(\tfrac{2}{3}\right) + 2\cdot\left(\tfrac{2}{3}\right)^2 + \cdots$$

first term $a = 2$

common ratio $r = \tfrac{2}{3}$

$$= \sum_{n=1}^{\infty} 2\cdot\left(\tfrac{2}{3}\right)^{n-1}$$

a r

Method 2: a = first term

$r = \dfrac{a_{n+1}}{a_n}$ = common ratio

for above, $a_n = 2^n \cdot 3^{1-n}$

so $$r = \frac{2^{(n+1)}\cdot 3^{1-(n+1)}}{2^n\cdot 3^{1-n}} = \frac{2^{n+1}}{2^n}\cdot\frac{3^1\cdot 3^{-n}\cdot 3^{-1}}{3^1\cdot 3^{-n}}$$

$$= 2\cdot 3^{-1} = \frac{2}{3}$$ ✓

Method 3: rewrite to "find" the $(n-1)$'s

$$\sum_{n=1}^{\infty} 2^n 3^{1-n} = \sum_{n=1}^{\infty} 2 \cdot 2^{n-1} \cdot 3^{-(n-1)}$$

$$= \sum_{n=1}^{\infty} 2 \cdot \frac{2^{n-1}}{3^{n-1}} = \sum_{n=1}^{\infty} 2 \cdot \left(\frac{2}{3}\right)^{n-1}$$

first term

$a = 2$

common ratio

$r = \frac{2}{3}$