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Remember: A sequence is an ordered *list of numbers.*

A Library of Common Sequences

Many sequences are combinations of easily described sequences.
Write out the first 5 terms of each common sequence.

Even Numbers:

$$\left\{2n\right\}_{n=1}^{\infty} = \{2\cdot 1, 2\cdot 2, 2\cdot 3, 2\cdot 4, 2\cdot 5, \dots\} = \{2, 4, 6, 8, 10, \dots\}$$

Odd Numbers:

$$\left\{2n-1\right\}_{n=1}^{\infty} = \{2\cdot 1-1, 2\cdot 2-1, 2\cdot 3-1, 2\cdot 4-1, 2\cdot 5-1, \dots\} = \{1, 3, 5, 7, 9$$ [redacted]

Alternating Signs:

$$\left\{(-1)^n\right\}_{n=1}^{\infty} = \{(-1)^1, (-1)^2, (-1)^3, (-1)^4, (-1)^5, \dots\} = \{-1, 1, -1, 1, -1, \dots\}$$

$$\left\{(-1)^{n+1}\right\}_{n=1}^{\infty} = \{(-1)^{1+1}, (-1)^{2+1}, (-1)^{3+1}, (-1)^{4+1}, (-1)^{5+1}, \dots\} = \{1, -1, 1, -1, 1, \dots$$

$$\left\{(-1)^{n-1}\right\}_{n=1}^{\infty} = \{(-1)^{1-1}, (-1)^{2-1}, (-1)^{3-1}, (-1)^{4-1}, (-1)^{5-1}, \dots\} = \{1, -1, 1, -1, 1, \dots\}$$

$$\left\{\cos(\pi n)\right\}_{n=1}^{\infty} = \{\cos(\pi), \cos(2\pi), \cos(3\pi), \cos(4\pi), \cos(5\pi)$$ [redacted] $$, \dots\}$$ [redacted]

$$= \{-1, 1, -1, 1, -1, \dots\}$$

[redacted]

Squares:

$$\left\{n^2\right\}_{n=1}^{\infty} = \{1^2, 2^2, 3^2, 4^2, 5^2, \dots\} = \{1, 4, 9, 16, 25, \dots\}$$

Powers:

$$\left\{2^n\right\}_{n=1}^{\infty} = \{2^1, 2^2, 2^3, 2^4, 2^5, \dots\} = \{2, 4, 8, 16, 32, \dots\}$$

For some sequences, simplifying the terms is *not* helpful.

Factorials: $\left\{\frac{2^n}{n!}\right\}_{n=1}^{\infty} = \left\{ 2, \frac{2\cdot 2}{1\cdot 2}, \frac{2\cdot 2\cdot 2}{1\cdot 2\cdot 3}, \dots \right\}$

Large powers: $\left\{\frac{1}{7^n}\right\}_{n=1}^{\infty} = \left\{ \frac{1}{7}, \frac{1}{7\cdot 7}, \frac{1}{7\cdot 7\cdot 7}, \dots \right\}$

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Extracting a Sequences' Definition

Sequences made by combining common sequences

Eg. Find a formula to define the sequence

$$\left\{\frac{3}{5}, \frac{-4}{25}, \frac{5}{125}, \frac{-6}{625}, \ldots\right\}$$

1. How can you generate the pattern of the sign (positive/negative)?

$\{1, -1, 1, -1, \ldots\}$ can be defined using $a_n = (-1)^{n-1}$

2. How can you generate the number on the top?

$\{\underset{a_1}{3}, \underset{a_2}{4}, \underset{a_3}{5}, \underset{a_4}{6}, \ldots\}$ can be defined using $a_n = (n+2)$

3. How can you generate the number on the bottom?

$\{5, 25, 125, 625, \ldots\}$ can be defined using $a_n = 5^n$

4. Putting it together, $a_n = \boxed{\left(\frac{(-1)^{n-1}(n+2)}{5^n}\right)}$

Sequences made by re-indexing common sequences

1. Consider the sequence $\{2, 3, 4, 5, \ldots\}$

   The n^{th} term in this sequence is the $(n+1)^{st}$ counting number.
   Remember that the n^{th} counting number is n. Therefore, $a_n = (n+1)$.

2. Consider the sequence $\{6, 8, 10, 12, \ldots\}$

   The n^{th} term in this sequence is the $(n+2)^{nd}$ even number.
   Remember that the n^{th} even number is $2n$. Therefore, $a_n = 2(n+2)$.

   Double check this by writing out $\left\{2(n+2)\right\}_{n=1}^{\infty} =$

3. Consider the sequence $\{5, 7, 9, 11, \ldots\}$

   The n^{th} term in this sequence is the $(n+2)^{nd}$ odd number.
   Remember that the n^{th} odd number is $2n-1$. Therefore, $a_n = 2(n+2) - 1 = 2n+3$

   Double check this by writing out $\left\{2n+3\right\}_{n=1}^{\infty} = \{2+3, 2\cdot2+3, 2\cdot3+3, \ldots\}$

   $= \{5, 7, 9, \ldots\}$

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Finding Limits of Sequences

Using tricks for functions

Sometimes the limit of the sequence (list of numbers) is the limit of the defining function.

1. Let $a_n = \dfrac{n^7+n^3+1}{n^6+4}$. What is the limit of the sequence (list of numbers) $\left\{a_n\right\}_{n=1}^{\infty}$?

$$\lim_{n\to\infty} \frac{n^7+n^3+1}{n^6+4} = \lim_{n\to\infty} \frac{n^7\left(1+\frac{1}{n^4}+\frac{1}{n^7}\right)}{n^6\left(1+\frac{4}{n^6}\right)} = \lim_{n\to\infty} \underbrace{n}_{\infty} \cdot \underbrace{\left(\frac{1+\frac{1}{n^4}+\frac{1}{n^7}}{1+\frac{4}{n^6}}\right)}_{\to 1} = \infty$$

2. Let $a_n = \dfrac{4^{n+1}}{9^n}$. What is the limit of the sequence (list of numbers) $\left\{a_n\right\}_{n=1}^{\infty}$?

$$\lim_{n\to\infty} \frac{4^{n+1}}{9^n} = \lim_{n\to\infty} 4\cdot\frac{4^n}{9^n} = \lim_{n\to\infty} 4\underbrace{\left(\frac{4}{9}\right)}_{\text{less than } 1}^n = 0$$

3. Let $a_n = \dfrac{e^n+e^{-n}}{e^{2n}}$. What is the limit of the sequence (list of numbers) $\left\{a_n\right\}_{n=1}^{\infty}$?

$$\lim_{n\to\infty} \frac{e^n+e^{-n}}{e^{2n}} = \lim_{n\to\infty} \frac{e^n\left(1+e^{-2n}\right)}{e^{2n}} = \lim_{n\to\infty} \frac{\overbrace{1+\overbrace{e^{-2n}}^{\to 0}}^{\to 1}}{\underbrace{e^n}_{\to\infty}} = 0$$

4. Let $a_n = \dfrac{(-1)^n\sqrt{n}}{n+1}$. What is the limit of the sequence (list of numbers) $\left\{a_n\right\}_{n=1}^{\infty}$?

$$\lim_{n\to\infty} \frac{(-1)^n\sqrt{n}}{n+1} = 0$$

by squeeze theorem

$$\frac{-\sqrt{n}}{n+1} \le \frac{(-1)^n\sqrt{n}}{n+1} \le \frac{\sqrt{n}}{n+1}$$

and $\displaystyle\lim_{n\to\infty} \frac{\sqrt{n}}{n+1} = \lim_{n\to\infty} \frac{\sqrt{n}}{n\left(1+\frac{1}{n}\right)} = \lim_{n\to\infty} \frac{1}{\underbrace{\sqrt{n}}_{\infty}\underbrace{\left(1+\frac{1}{n}\right)}_{1}} = 0$

By thinking carefully

Sometimes the limit of the sequence (list of numbers) is **NOT** the limit of the defining function.

1. If $a_n = \sin(n)$, then $\{\sin(1), \sin(2), \sin(3), \sin(4), \ldots\}$ diverges because it oscillates between -1 and 1, taking on *many different* values.

   To see this, use your calculator to evaluate $\sin(n)$ for $n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$.

2. If $a_n = \sin(2\pi n)$, then $\{\sin(2\pi), \sin(4\pi), \sin(6\pi), \sin(8\pi), \ldots\}$ converges because a_n is always *equal* to 1.

3. If $a_n = \sin(\pi n)$, then $\{\sin(\pi), \sin(2\pi), \sin(3\pi), \sin(4\pi), \ldots\}$ diverges because it alternates between *two* values, -1 and 1.

In conclusion:

Always do a "sanity-check" for your answer. If you plugged in actual counting numbers, what would the sequence (list of numbers) do? If you aren't certain, actually *do* plug in numbers!