Eg: $\int_{-\infty}^{\infty} x \, dx$

(it is helpful to pick $a = 0$)

(< 5 mm)

$$= \int_{-\infty}^{0} x \, dx + \int_{0}^{\infty} x \, dx$$

check yourself: diverges to $-\infty$; diverges to ∞

$\Rightarrow$ the original integral DIVERGES.

Eg: $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$

(again <u>helpful</u> to pick $a=0$)

$$= \int_{-\infty}^{0} \frac{1}{1+x^2}\,dx + \int_{0}^{\infty} \frac{1}{1+x^2}\,dx$$

$$= \lim_{t\to-\infty} \left[\int_{t}^{0} \frac{1}{1+x^2}\,dx \right] + \lim_{t\to\infty} \left[\int_{0}^{t} \frac{1}{1+x^2}\,dx \right]$$

• • •

$$= \lim_{t\to-\infty} \left(\tan^{-1}(0) - \tan^{-1}(t) \right) + \lim_{t\to\infty} \left(\tan^{-1}(t) - \tan^{-1}(0) \right)$$

$\tan^{-1}(0) = 0$, $\tan^{-1}(t) \to -\frac{\pi}{2}$, $\tan^{-1}(t) \to \frac{\pi}{2}$, $\tan^{-1}(0) = 0$

$$= -\left(\frac{-\pi}{2}\right) + \frac{\pi}{2} = \pi .$$