Quiz #4 Math 1131 March 31, 2014

Name: _____ Section: _____

You have 12 minutes to complete the quiz. Please **show all work**, and then **circle your answer.**

1. (5 points) Suppose that an outbreak of smallpox in Waterbury grows at a rate proportional to its size. Suppose that the outbreak begins with 5 sick people arriving on an airplane, and that 500 people are ill after 2 days have passed.

   Find an equation for the population as a function of t in days since the outbreak began.

1pt $P(t) = P_0 \cdot e^{kt}$

$500 = P(2) = 5 \cdot e^{k \cdot 2}$

$100 = e^{k \cdot 2}$

$\ln(100) = 2k$

$k = \frac{\ln(100)}{2}$

2pt

2pt $P(t) = 5 \cdot e^{\left(\frac{\ln(100)}{2} t\right)}$

2. (5 points) Compute the following limit

$$\lim_{x \to \infty} x \cdot \ln\left(1 + \frac{2}{x}\right)$$

$\infty \cdot 0$ $\leftarrow$ 1 pt

1pt $= \lim_{x \to \infty} \frac{\ln\left(1 + \frac{2}{x}\right)}{\frac{1}{x}}$ ($\ln(\cdot) \to 0$, $\frac{1}{x} \to 0$)

derivative of top: $\frac{d}{dx} \ln(1 + 2x^{-1}) = \frac{1}{1 + 2x^{-1}} \cdot (-2x^{-2})$

deriv of bottom: $\frac{d}{dx}(x^{-1}) = -1 \cdot x^{-2}$

2pt $\overset{H}{=} \lim_{x \to \infty} \frac{\frac{1}{1 + \frac{2}{x}} \cdot \frac{-2}{x^2}}{-\frac{1}{x^2}} \cdot \frac{-x^2}{-x^2}$

$= \lim_{x \to \infty} \frac{\frac{1}{1 + \frac{2}{x}} \cdot 2}{1} = 2$

1pt