1134

Quiz #3 Math 1131 March 14, 2014

Name: Key Section: ______________

You have 15 minutes to complete the quiz. Please **show work** and then **circle your answer.**

1. (2 points) Let $f(x) = \dfrac{x^2+1}{x}$. Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[(x^2+1)x^{-1}\right]$$

$$= \frac{d}{dx}\left[x + x^{-1}\right]$$

$$= 1 - x^{-2}$$

2pt

Alternative method

(quotient Rule)

$$f'(x) = \frac{x \cdot \frac{d}{dx}[x^2+1] - (x^2+1)\frac{d}{dx}[x]}{x^2}$$

$$= \frac{x \cdot 2x - (x^2+1)}{x^2}$$

$$= \frac{x^2-1}{x^2}$$

2. (3 points) Suppose that $\sqrt{y+1} = x + y$. Find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left[(y+1)^{\frac{1}{2}}\right] = \frac{d}{dx}[x+y]$$

$$\frac{1}{2}(y+1)^{-\frac{1}{2}} \cdot \frac{d}{dx}[y+1] = 1 + \frac{dy}{dx}$$

1pt

$$\frac{1}{2\sqrt{y+1}} \cdot \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{1}{2\sqrt{y+1}} \cdot \frac{dy}{dx} - \frac{dy}{dx} = 1$$

$$\frac{dy}{dx}\left[\frac{1}{2\sqrt{y+1}} - 1\right] = 1 \implies \frac{dy}{dx} = \frac{1}{\frac{1}{2\sqrt{y+1}} - 1}$$

2pt

1202

Name: ______________________ Section: ______________________

3. (2 points) Let $f(x) = \ln\left(\frac{e^x}{x}\right)$. Find $f'(x)$

$$f'(x) = \frac{d}{dx}\left[\ln\left(\frac{e^x}{x}\right)\right]$$

$$= \frac{d}{dx}\left[\ln(e^x) - \ln(x)\right]$$

1pt $$= \frac{d}{dx}\left[x - \ln(x)\right]$$

1pt $$= 1 - \frac{1}{x}$$

OR

$$f'(x) = \frac{d}{dx}\left[\ln\left(\frac{e^x}{x}\right)\right]$$

$$= \frac{1}{\frac{e^x}{x}} \cdot \frac{d}{dx}\left[\frac{e^x}{x}\right]$$

$$= \left(\frac{x}{e^x}\right)\left(\frac{x \cdot e^x - e^x \cdot 1}{x^2}\right)$$

$$= \frac{x}{x^2} \cdot \left(\frac{x \cdot e^x - e^x}{e^x}\right)$$

$$= \frac{1}{x} \cdot \frac{\cancel{e^x}(x-1)}{\cancel{e^x}}$$

$$= \frac{x-1}{x} = 1 - \frac{1}{x}$$

4. (3 points) Let $y = (1-x)^{5x}$. Find $y' = \frac{dy}{dx}$.

$$y = (1-x)^{5x}$$

introduce ln

1pt

$$\ln(y) = \ln\left((1-x)^{5x}\right)$$

$$\ln(y) = 5x \cdot \ln(1-x)$$

take derivatives

$$\frac{d}{dx}\left[\ln(y)\right] = \frac{d}{dx}\left[5x \cdot \ln(1-x)\right]$$

$$\frac{1}{y} \cdot y' = 5x \cdot \frac{d}{dx}\left[\ln(1-x)\right] + \ln(1-x) \cdot \frac{d}{dx}[5x]$$

$f(u) = \ln(u) \Rightarrow f'(u) = \frac{1}{u}$

1pt $$\frac{1}{y} \cdot y' = 5x \cdot \frac{1}{1-x} \cdot \underbrace{\frac{d}{dx}[1-x]}_{=-1} + \ln(1-x) \cdot 5$$

solve for y'

$$y' = y\left(\frac{-5x}{1-x} + 5 \cdot \ln(1-x)\right) = (1-x)^{5x} \cdot \left(\frac{-5x}{1-x} + 5\ln(1-x)\right)$$

1pt

1205