Instructions:

- This exam contains 11 pages. When we begin, check you have *one* of each page.
- You will have 70 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.**
  In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:
By writing my name below, I agree that all the work
which appears on this exam is entirely my own.
I will not look at other peoples' work,
and I will not communicate with anyone else about the exam.
I will not use any calculators, notes, etc.
I understand that violating the above carries *serious consequences*,
both moral and academic.

Printed Name: Key Signature: ____________

Section: ____________

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	10	10	10	10	10	6	12	12	100
Score:											

1. (a) [5 points] Let $f(x) = \sqrt{\tan(x)}$. Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left(\left(\tan(x)\right)^{\frac{1}{2}}\right)$$

$$= \frac{1}{2}\cdot\left(\tan(x)\right)^{-\frac{1}{2}}\cdot\frac{d}{dx}\left(\tan(x)\right) \quad 3\text{ pt}$$

$$= \frac{1}{2}\cdot\frac{1}{\sqrt{\tan(x)}}\cdot\sec^2(x) \quad 2\text{ pt}$$

$$= \frac{\sec^2(x)}{2\sqrt{\tan(x)}}$$

(a) ____________

(b) [5 points] Let $f(x) = x\ln(x^2)$. Find $f''(x)$.

$f(x) = x\cdot 2\ln(x)$ by log law.

$$f'(x) = \frac{d}{dx}\left(2x\cdot\ln(x)\right)$$

$$= 2x\cdot\frac{1}{x} + 2\cdot\ln(x)$$

$$= 2 + 2\cdot\ln(x)$$

3 pt.

$$f''(x) = 2\cdot\frac{1}{x} = \frac{2}{x}$$

2 pt

(b) ____________

1919

2. (a) [5 points] Let $f(x) = (x^2+3)^x$. Find $f'(x)$.

$$y = (x^2+3)^x$$

$$\ln(y) = \ln\left((x^2+3)^x\right)$$

2pt

$$\ln(y) = x \cdot \ln(x^2+3)$$

$$\frac{1}{y}\cdot y' = x \cdot \frac{d}{dx}\left(\ln(x^2+3)\right) + \ln(x^2+3)\cdot\frac{d}{dx}(x)$$

2pt

$$\frac{1}{y} y' = x \cdot \frac{1}{x^2+3}\cdot 2x + \ln(x^2+3)$$

1pt

$$y' = y\left(\frac{2x^2}{x^2+3} + \ln(x^2+3)\right) = \boxed{(x^2+3)^x\left(\frac{2x^2}{x^2+3} + \ln(x^2+3)\right)}$$

(a) ____________

(b) [5 points] Let $3y^2 = \sin(y) + x^2$. Find $\frac{dy}{dx}$.

$$\frac{d}{dx}(3y^2) = \frac{d}{dx}\left(\sin(y) + x^2\right)$$

2pt

$$3\cdot 2y\cdot y' = \cos(y)\cdot y' + 2x$$

$$6y\cdot y' - \cos(y)\cdot y' = 2x$$

2pt

$$y'(6y - \cos(y)) = 2x$$

1pt

$$y' = \frac{2x}{6y - \cos(y)}$$

2.5/5
if you lose one y'
(it changes the problem a lot)
but otherwise correct

(b) ____________

1921

3. [10 points] Suppose that you begin with 200 kg of a radioactive substance. Suppose also that the substance has a half life of 6 years.

(a) Find a formula for the amount of radioactive substance remaining after t years.

1pt $P(t) = P_0 e^{kt} = 200 e^{kt}$

Know $P(6) = 100 = 200 e^{k6}$

$$\frac{1}{2} = e^{k6}$$

$$\ln\left(\tfrac{1}{2}\right) = k \cdot 6$$

2pt $k = \dfrac{\ln\left(\frac{1}{2}\right)}{6}$

2pt so $P(t) = 200\, e^{\frac{\ln\left(\frac{1}{2}\right)}{6} t}$

(b) How long must you wait until only 2 kg of the radioactive substance remians?

want t s.t.

2pt $2 = 200 \cdot e^{\frac{\ln\left(\frac{1}{2}\right)}{6} t}$

1pt $\dfrac{1}{100} = e^{\frac{\ln\left(\frac{1}{2}\right)}{6} t}$

$$\ln\left(\frac{1}{100}\right) = \frac{\ln\left(\frac{1}{2}\right)}{6} t$$

2pt $t = \dfrac{6 \cdot \ln\left(\frac{1}{100}\right)}{\ln\left(\frac{1}{2}\right)}$

$$= \frac{6 \cdot \ln(100^{-1})}{\ln(2^{-1})} = \frac{6 \cdot \cancel{(-1)} \cdot \ln(100)}{\cancel{(-1)} \cdot \ln(2)} = \frac{6 \cdot \ln(100)}{\ln(2)}$$

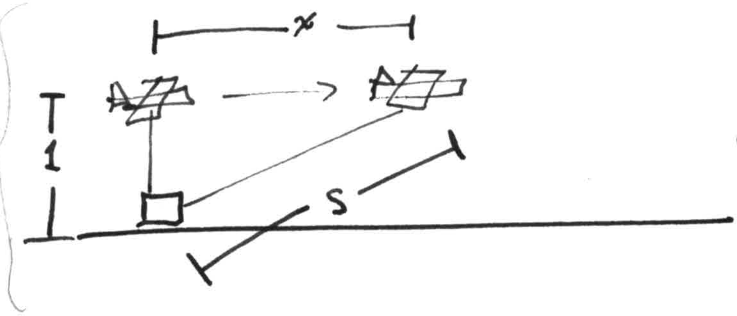
4. [10 points] An airplane flies directly over a radar station, at a constant altitude of 1 mi above the ground. A little while later, the radar station measures that (a) the distance between the plane and radar station equals $\sqrt{10}$ mi and (b) that the distance between the plane and radar station is increasing at a rate of 300 mi/hr.

What is the ground speed of the airplane at the time of the second measurement?

You must show all work.

want $\frac{dx}{dt}$ when $s = \sqrt{10}$ and $\frac{ds}{dt} = 300$

2pt



① Relate the Variables

1pt

$$1^2 + x^2 = s^2$$

② Relate the Rates

3pt

$$\frac{d}{dt}\left(1^2 + x^2\right) = \frac{d}{dt}\left(s^2\right)$$

$$2x \cdot x' = 2 s s'$$

③ Answer the question

2pt

when $s = \sqrt{10}$

$$1 + x^2 = \left(\sqrt{10}\right)^2 = 10$$

$$x^2 = 9$$

$$x = 3$$

~~Know: $2xx' = 2ss'$~~

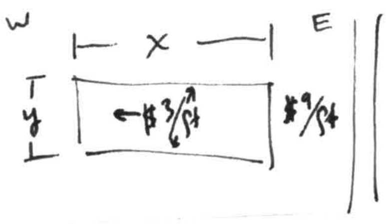
so $2 \cdot 3 \cdot x' = 2 \cdot \sqrt{10} \cdot 300$

2pt →

$$x' = 100\sqrt{10}$$

5. 10 points] Suppose you want to build a fenced-in rectangular playground with a road on the east side. Suppose that the fence on the east side costs \$9 per foot, and that the fence on the other sides costs \$3 per foot. If you have \$120, what is the largest area that you can enclose?

You must show all work, including verifying that this area is a maximum.



Constraint

$120 = 9(y) + 3(x + y + x)$

setup 3 pt

$120 = 9y + 6x + 3y$

$120 = 12y + 6x$

$12y = 120 - 6x$

$y = 10 - \frac{1}{2}x$

2pt

Maximize area

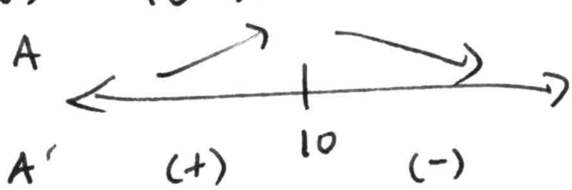
$A = xy$

$A(x) = x(10 - \frac{1}{2}x) = 10x - \frac{x^2}{2}$

2pt

$A'(x) = 10 - x$

1pt



1pt area max when $x = 10$

$y = 10 - \frac{1}{2}(10) = 5$

1pt max area $= 50$.

1931

6. 10 [redacted] points] Find numbers x and y with $yx = 8$ such that $F = x^2 + 2y$ is minimized. You must show all work, including verifying that F is minimized.

Minimize

$F = x^2 + 2y$

constraint

$yx = 8$

$y = \frac{8}{x}$ 2pt

$F(x) = x^2 + \frac{2 \cdot 8}{x}$

$F(x) = x^2 + \frac{16}{x}$ 2pt

2pt → $F'(x) = 2x - \frac{16}{x^2}$

Crit #

$F'(x)$ ~~=2~~ DNE when $x=0$

$F'(x) = 0$ when $2x = \frac{16}{x^2}$

$2x^3 = 16$

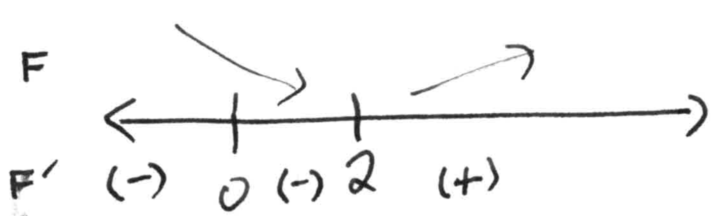
$x^3 = 8$

$x = 2$

2pt →

−1 pt if missing 0

Check: 1pt



F is minimized when $x = 2$ & $y = \frac{8}{2} = 4$

Answer 1pt

1933

7. [10 points] (a) Find the line tangent to $f(x) = \sqrt{x}$ at $a = 100$.

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{2 \cdot 10} = \frac{1}{20}$$

$$f(100) = \sqrt{100} = 10$$

5pt

tangent line to $\sqrt{x}$ at 100 $= L(x) = \frac{1}{20}(x - 100) + 10$

(b) Use linear approximations to estimate $\sqrt{98}$. Simplify completely and give your answer as a decimal.

5 pt

Because $98 \approx 100$

$$\sqrt{98} \approx L(98) = \frac{1}{20}(98 - 100) + 10$$

$$= \frac{1}{20}(-2) + 10$$

$$= \frac{-1}{10} + 10$$

$$= 9.9$$

8. [6 points] Use L'Hospitals Rule to compute the following limit:

$$\lim_{x\to\infty} \underbrace{4x}_{\infty}\, \underbrace{\sin\left(\underbrace{\frac{1}{x}}_{0}\right)}_{0}$$

type $0\cdot\infty$

2pt

$$= \lim_{x\to\infty} \frac{4\cdot \sin\left(\frac{1}{x}\right)}{\frac{1}{x}} \quad \begin{matrix}\to 0\\ \to 0\end{matrix}$$

2pt

$$\overset{H}{=} \lim_{x\to\infty} \frac{4\cdot\cos\left(\frac{1}{x}\right)\cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}}$$

$$= \lim_{x\to\infty} \underbrace{4\cdot\cos\left(\underbrace{\frac{1}{x}}_{0}\right)}_{1}$$

2pt

$$= 4$$

9. [12 points] Let $f(x) = x^3 - 3x^2 + 5$

Find the following. If a requested quantity doesn't exist, answer DNE.

(a) The intervals where $f(x)$ is increasing/decreasing. Identify which is which.

1 → $f'(x) = 3x^2 - 6x = 3x(x-2)$

1 → f: $\nearrow$ 0 $\searrow$ 2 $\nearrow$

f': $(-)(-)$ 0 $(+)(-)$ 2 $(+)(+)$

$(+)$ $(-)$ $(+)$

3 → inc: $(-\infty, 0) \cup (2, \infty)$ dec: $(0, 2)$

(b) The intervals where $f(x)$ is concave up/down. Identify which is which.

1 → $f''(x) = 6x - 6 = 6(x-1)$

1 → f: ⌒ 1 ⌣

f'': $(-)$ 1 $(+)$

1 → concave down: $(-\infty, 1)$

1 → concave up: $(1, \infty)$

(c) The x value(s) of the local maxima and local minima of f. Identify which is which.

1 → local max: 0

1 → local min: 2

(d) The x value(s) of the inflection points of f.

1 → inflection pt at 1

1942

10. [12 points] Anti-derivatives

(a) Compute the general antiderivative of $f(x) = \left(2x+1\right)\left(4x - \dfrac{1}{x}\right)$.

$$f(x) = 8x^2 - 2x\cdot\frac{1}{x} + 4x - \frac{1}{x}$$

$$= 8x^2 - 2 + 4x - \frac{1}{x}$$

2pt

max 2pt if incorrectly find antideriv. w/o FOILing first

$$F(x) = \frac{8x^3}{3} - 2x + \frac{4x^2}{2} - \ln|x| + C$$

$$\boxed{= \frac{8}{3}x^3 - 2x + 2x^2 - \ln|x| + C}$$

2pt

(b) Suppose that $f''(x) = 12x^2 - 6x + 2$, that $f(1) = 1$, and that $f'(1) = 2$. Find $f(x)$.

$$f'(x) = 12\frac{x^3}{3} - 6\frac{x^2}{2} + 2x + C$$

$$= 4x^3 - 3x^2 + 2x + C$$

2pt

$$f'(1) = 2 = 4\cdot 1^3 - 3\cdot 1^2 + 2\cdot 1 + C$$

$$2 = 4 - 3 + 2 + C$$

$$C = -1$$

$$f'(x) = 4x^3 - 3x^2 + 2x - 1$$

2pt

$$f(x) = 4\cdot\frac{x^4}{4} - 3\frac{x^3}{3} + \frac{2x^2}{2} - x + D$$

$$= x^4 - x^3 + x^2 - x + D$$

2pt

$$f(1) = 1 = \cancel{1^4 - 1^3} + 1^2 - 1 + D$$

$$D = 1$$

2pt

$$\boxed{f(x) = x^4 - x^3 + x^2 - x + 1}$$