Instructions:

- This exam contains 10 pages. When we begin, check you have *one* of each page.
- You will have 70 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.** In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:
By writing my name below, I agree that:
All the work which appears on this exam is entirely my own.
I will not look at other peoples' work,
and I will not communicate with anyone else during the exam.
I will not use any calculators, notes, etc.
I understand that violating the above carries *serious consequences*,
both moral and academic.

Printed Name: Key Signature: ______

Section: ______

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	12	10	4	15	8	12	15	10	14	100
Score:										

1. Exponential and logarithmic properties

(a) [3 points] Express the following as a single logarithm, simplifing where possible.

$$2\ln(x) + 5\ln(y) - 3\ln(x)$$

$$= \ln(x^2) + \ln(y^5) - \ln(x^3)$$

1pt

$$= \ln\left(x^2 \cdot y^5\right) - \ln(x^3)$$

$$= \ln\left(\frac{x^2 \cdot y^5}{x^3}\right) = \boxed{\ln\left(\frac{y^5}{x}\right)}$$

1pt

1pt

(a) ______________________

(b) [3 points] Solve for x:

$$6 \cdot e^{1-x} = 3$$

$$e^{1-x} = \frac{3}{6} = 1/2$$ ← 1pt

$$\ln(e^{1-x}) = \ln\left(\tfrac{1}{2}\right)$$

$$1 - x = \ln\left(\tfrac{1}{2}\right)$$ ← 1pt

$$1 = \ln\left(\tfrac{1}{2}\right) + x$$

(b) $\boxed{x = 1 - \ln\left(\tfrac{1}{2}\right)}$ ← 1pt $= 1 + \ln\left(\left(\tfrac{1}{2}\right)^{-1}\right)$

$= 1 + \ln(2)$

(c) [3 points] Let $f(x) = x^4 + x^2 + 1$ and $g(x) = \sqrt{x}$. Write $f\Big(g(x)\Big)$ and simplify.

$$f\big(g(x)\big) = f(\sqrt{x}) = (\sqrt{x})^4 + (\sqrt{x})^2 + 1$$ ← 1pt

$\boxed{f\Big(g(x)\Big) = x^2 + x + 1}$ ← 2pt

(d) [3 points] Find functions $f(x)$ and $g(x)$ so that $f\Big(g(x)\Big) = \sqrt{x^2+1}$.

check:

$f\big(g(x)\big) = f(x^2+1)$

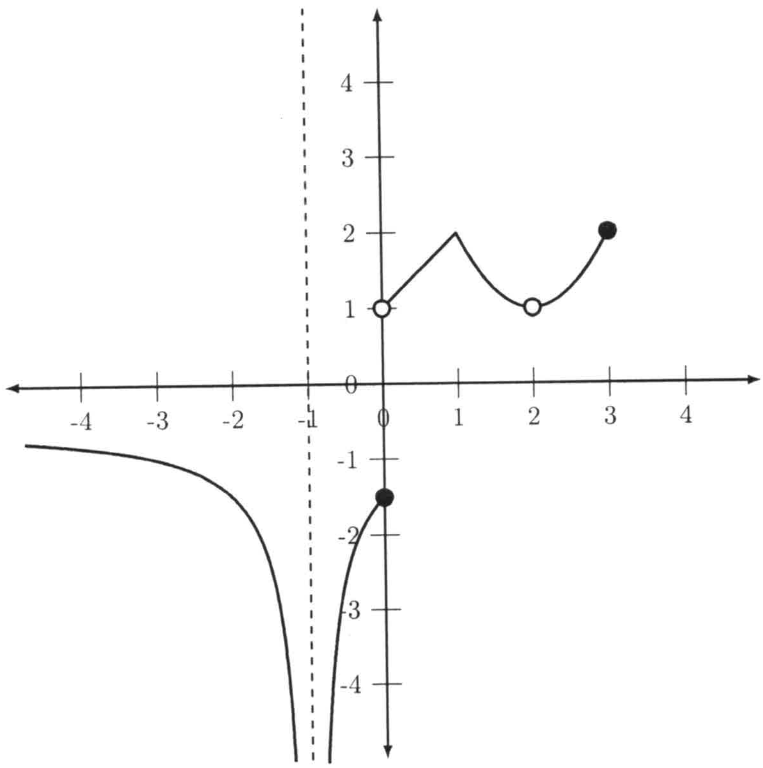
$= \sqrt{x^2+1}$ ✓

outside = $f(x) =$ $\sqrt{x}$

inside = $g(x) =$ $x^2 + 1$

3pt

2. Suppose that $f(x)$ is defined using the following graph.



(a) [3 points] Find the values of x where f is not continuous.

-1, 0, 2

1 pt each

(-1 for extra incorrect.)

(b) [3 points] Write the set of x where f is continuous in interval notation.

$(-\infty, -1) \cup (-1, 0) \cup (0, 2) \cup (2, 3]$

2pt correct intervals

1pt correct brackets

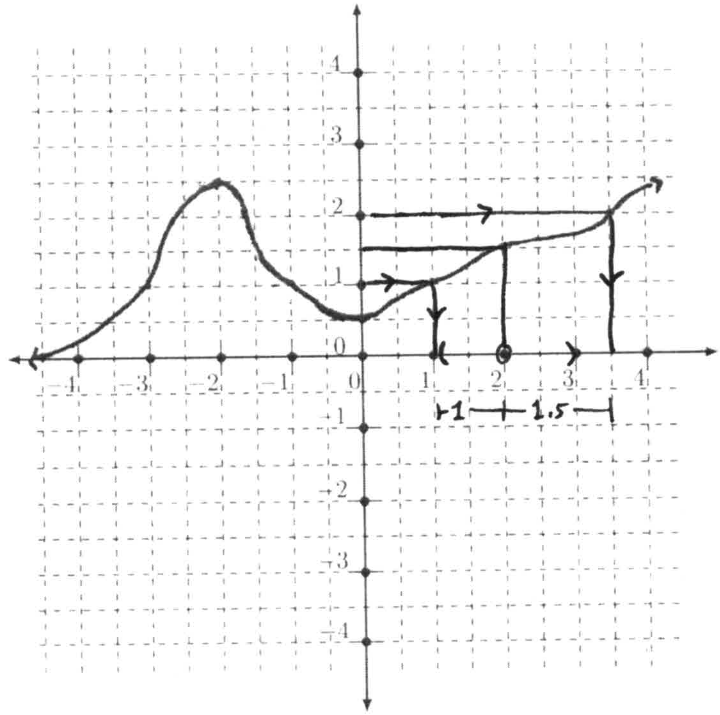
(c) [4 points] Where is $f(x)$ not differentiable? That is, for what x is $f'(x)$ undefined?

at -1, 0, 1, 2

1 pt each

3. Suppose that $f(x)$ is defined using the graph:

3 pt



(a) [4 points] How close must x be to 2 to ensure that $f(x)$ is within 0.5 units of 1.5? You must support your answer **by what you draw** in the figure.

1 pt

If x is within 1 unit of 2,

Then $f(x)$ is within 0.5 units of 1.5.

<u>aka:</u>

If $0<|x-2|<1$,

then $|f(x)-1.5|<0.5$

4. Evaluate the following limits. Be sure to **show all work.**

(a) [5 points] $\lim_{x\to 1^+} \frac{x^2+4x-5}{x^2-1}$ ← cannot plug in 1

2pt ⇒ $= \lim_{x\to 1^+} \frac{\cancel{(x-1)}(x+5)}{(x+1)\cancel{(x-1)}}$

$= \lim_{x\to 1^+} \frac{(x+5)}{(x+1)}$ ← continuous at $x=1$ ⇒ can plug in 1

3pt ⇒ $= \frac{1+5}{1+1} = \frac{6}{2} = 3$

(a) 3

(b) [5 points] $\lim_{x\to 1^+} \frac{x^2+3x+2}{x^2-1}$ ← cannot plug in 1

2pt ⇒ $= \lim_{x\to 1^+} \frac{(x+2)\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}$

$= \lim_{x\to 1^+} \frac{(x+2)^{\to 3}}{(x-1)}$ ← $\boxed{\text{cannot plug in 1}}$

1pt: as $x \to 1^+$ → $x-1$ is ① positive ② small; so $\frac{3}{x-1}$ is ① positive ② big

$= \infty$

(b) ∞ ⇐ 2pt

(c) [5 points] $\lim_{x\to 1^+} \frac{x^2+3x+2}{x^2+1}$ ← continuous at 1 ⇒ can plug in 1 ⇐ 1pt

$= \frac{1+3+2}{1+1} = \frac{6}{2} = 3$

3 ⇐ 4pt

(c)

5. Evaluate the following limits. ***Remember*** to **show work.**

(a) [4 points] $\displaystyle\lim_{x\to\infty} \frac{2x^2 - x + 10}{6x^3 - 7x^2 + 2}$

$$= \lim_{x\to\infty} \frac{x^2\left(2 - \frac{1}{x} + \frac{10}{x^2}\right)}{x^3\left(6 - \frac{7}{x} + \frac{2}{x^3}\right)}$$

2pt (REQUIRED)

$$= \lim_{x\to\infty} \frac{\left(2 - \overset{\to 0}{\frac{1}{x}} + \overset{\to 0}{\frac{10}{x^2}}\right)}{\underbrace{x}_{\infty} \cdot \underbrace{\left(6 - \frac{7}{x} + \frac{2}{x^3}\right)}_{6}} \quad \begin{matrix} \to 2 \\ \to \infty \end{matrix}$$

2pt

$$= 0$$

$\boxed{\text{1 pt max for guesses}}$

(a) $\boxed{0}$

(b) [4 points] $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{x^6 + 7x + 5}}{7x^3 - 2x^2 + x}$

$$= \lim_{x\to-\infty} \frac{\sqrt{x^6\left(1 + \frac{7}{x^5} + \frac{5}{x^6}\right)}}{x^3\left(7 - \frac{2}{x} + \frac{1}{x^2}\right)}$$

2 pt (REQUIRED)

$$= \lim_{x\to-\infty} \frac{\cancel{x^3}\sqrt{1 + \overset{\to 0}{\frac{7}{x^5}} + \overset{\to 0}{\frac{5}{x^6}}}}{\cancel{x^3}\left(7 - \underset{0}{\frac{2}{x}} + \underset{\to 0}{\frac{1}{x^2}}\right)}$$

-2 pt if you distributed the $\sqrt{}$

2 pt

$$= \frac{\sqrt{1}}{7}$$

$\boxed{\text{1 pt max for guesses}}$

(b) $\boxed{\frac{1}{7}}$

6. The definition and meaning of the derivative.

(a) [4 points] Write down the limit definition of the derivative of the function $f(x)$.

4pt

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\boxed{\text{-2points if you omit the limit}}$

(b) [4 points] Explain the graphical meaning of $f'(a)$ with words and with a sketch.

words: $f'(a)$ is the slope of $f(x)$ at a

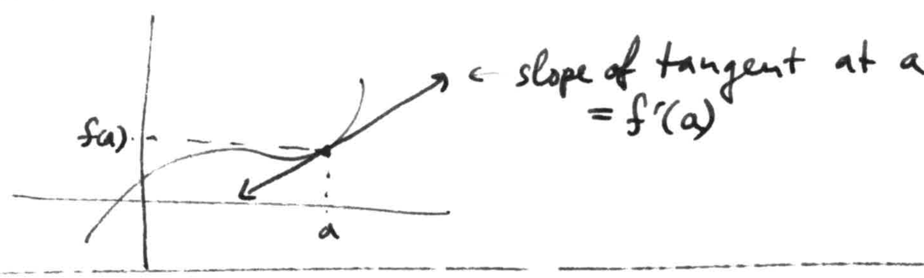
or

2pt

$f'(a)$ is the instantaneous rate of change of $f(x)$ at a

sketch:

2pt



(c) [4 points] Let $f(x) = x^2 + 2x$. Find $f'(2)$ using the *limit definition* of the derivative.

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$f(2+h) = (2+h)^2 + 2(2+h)$

$= 4 + 4h + h^2 + 4 + 2h$

$= 8 + 6h + h^2$

2pt

$$= \lim_{h \to 0} \frac{(\cancel{8} + 6h + h^2) - (\cancel{4+4})}{h}$$

1pt

$$= \lim_{h \to 0} \frac{6h + h^2}{h}$$

← 1pt

$$= \lim_{h \to 0} [6 + h] = \boxed{6 = f'(2)}$$

7. From this question onward, you may use the derivative rules. Show major steps for full credit.

(a) [5 points] Let $f(x) = 12e^x - 3x^5 + 2x$. Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[12e^x - 3x^5 + 2x\right]$$

$$= 12 \cdot \frac{d}{dx}[e^x] - 3 \cdot \frac{d}{dx}[x^5] + 2 \cdot \frac{d}{dx}[x]$$

$$\boxed{f'(x) = 12 \cdot e^x - 15x^4 + 2}$$

5pt

(a) ______________________

(b) [5 points] Compute the derivative of

$$f(x) = \sqrt{x} \cdot \cos(x)$$

$(fg)' = fg' + gf'$

$$f'(x) = \frac{d}{dx}\left[(x)^{\frac{1}{2}} \cdot \cos(x)\right] = x^{\frac{1}{2}} \cdot \frac{d}{dx}[\cos(x)] + \cos(x) \cdot \frac{d}{dx}\left[x^{\frac{1}{2}}\right]$$

2 pt

$$= x^{\frac{1}{2}} \cdot (-\sin(x)) + \cos(x) \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}}$$

3pt

$$= \boxed{-\sqrt{x} \cdot \sin(x) + \frac{\cos(x)}{2\sqrt{x}}}$$

(b) ______________________

(c) [5 points] Compute $\frac{d}{dx}\left[\frac{\sin(x)}{e^x}\right]$

$\left(\frac{t}{b}\right)' = \frac{bt' - tb'}{b^2}$

3pt

$$= \frac{e^x \cdot \frac{d}{dx}[\sin(x)] - \sin(x) \cdot \frac{d}{dx}[e^x]}{(e^x)^2}$$

$$= \frac{e^x \cdot \cos(x) - e^x \cdot \sin(x)}{e^x \cdot e^x}$$

2pt

$$= \frac{\cancel{e^x}(\cos(x) - \sin(x))}{\cancel{e^x} \cdot e^x}$$

(c) $\frac{\cos(x) - \sin(x)}{e^x}$

8. Let $f(x)$ be the function

$$f(x) = \frac{x}{x+2}$$

(a) [5 points] Find the *slope* of the tangent to $f(x)$ at the point with $x = 2$.

[slope of tangent at 2] = $f'(2)$

$\left(\frac{t}{b}\right)' = \frac{bt' - tb'}{b^2}$

$$f'(x) = \frac{d}{dx}\left[\frac{x}{x+2}\right] = \frac{(x+2)\cdot 1 - x\cdot 1}{(x+2)^2}$$

3 pt

$$= \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2}$$

$$f'(2) = \frac{2}{(2+2)^2} = \frac{2}{16} = \frac{1}{8}$$

2 pt

(a) $\boxed{m = \frac{1}{8}}$

(b) [5 points] Find the *equation* of tangent line to $f(x)$ at the point with $x = 2$.

$y = m(x - x_1) + y_1$

2pt

$x = 2$

$m = \frac{1}{8}$

$y_1 = f(2) = \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$

$$\boxed{y = \frac{1}{8}(x-2) + \frac{1}{2}}$$

3 pt

$= \frac{x}{8} + \frac{1}{4}$

(b) ____________________

9. Let $f(x)$ be the function

$$f(x) = \frac{x+3}{x} = \frac{x}{x} + \frac{3}{x} = 1 + \frac{3}{x} = 1 + 3\cdot x^{-1}$$

(a) [5 points] Find a formula for $f'(x)$.

$\left(\frac{t}{b}\right)' = \frac{bt' - tb'}{b^2}$

$$f'(x) = \frac{d}{dx}\left[\frac{x+3}{x}\right] = \frac{x\cdot 1 - (x+3)\cdot 1}{x^2} = \frac{x - x - 3}{x^2}$$

$$f'(x) = \frac{-3}{x^2}$$

OR

$$f'(x) = \frac{d}{dx}\left[1 + 3x^{-1}\right] = 0 + 3\cdot(-1)\cdot x^{-2} = \frac{-3}{x^2}$$

(a) $\boxed{f'(x) = \frac{-3}{x^2}}$

5pt

(b) [4 points] Find the tangent line to f at the point [redacted].

(1, 2) ← typo in question

(1, 4) ← actual y-value

2pt - line with y-value

$$y = m(x - x_1) + y_1$$

$x_1 = 1$

$y_1 = 2$

$$m = f'(1) = \frac{-3}{(1)^2} = -3$$

$$y = -3(x-1) + 2 = -3x + 5$$

zero points if slope is not a #

slope cannot have an x in it

(b) $\boxed{y = -3(x-1) + 2 \ \boxed{\text{OR}}\ y = -3x + 5}$

2pt

(c) [5 points] Find a formula for $f''(x)$.

$$f''(x) = \frac{d}{dx}\left[f'(x)\right] = \frac{d}{dx}\left[\frac{-3}{x^2}\right] = \frac{d}{dx}\left[(-3)\,x^{-2}\right] = (-3)(-2)\,x^{-3} = 6x^{-3}$$

2pt

(c) $\boxed{f''(x) = \frac{6}{x^3}}$

3pt