Name: _____ Section: _____

Principles of Problem Solving [1]

1. Understand the problem: Read the problem carefully.
   (a) What is *the unknown*? What are we optimizing?
   (b) What are the *data*? What descriptions and restrictions are we given?
   (c) If possible, draw a technical sketch (see below).
   (d) Introduce notation that allows you to use the data to write an equation for the unknown.
2. Think of a plan: find the function to optimize.
   (a) Use the data to write an equation for the unknown in terms of the other variables.
   (b) Find equations linking the other variables.
   (c) Express the unknown as a function of a **single** variable.
3. Carry out the plan: find the absolute minimum/maximum.
   (a) Find the derivative of the unknown, and find the critical points.
   (b) Find the min/max value either by (i) using the first derivative test for absolute extrema, or (ii) by plugging in values from a closed interval.
4. Look back
   (a) Check your work! Does your answer make sense?
   (b) Can you use this result or method for another problem?

Principles of Sketching[2]

There are four basic elements of a sketch

1. **The Drawing:** Sketch the physical objects being described. Try to match the scale and relations between things.
2. **Annotations:** Add names, labels, and explanatory notes.
   - Label quantites that can change with *letters*. If a quantity (length, angle, etc) *cannot* change, you can label the drawing with its value.
   - You might also want to add additional lines to create a shape like a triangle, which can be used along with trigonometry or the Pythagorean theorem.
3. **Arrows:** Draw arrows to indicate motion. Once drawn, these arrows can often help you find out where to fill in the missing lines to create a triangle.
4. **Notes:** Next to your drawing, write down any formulas that may be useful for relating the relevant quantities. Common examples are area, volume, trig, simila, and distance formulas. You may also use facts about similar triangles.

[1] Adapted from Polya's *How To Solve It* and Stewart's *Calculus* 7e
[2] Adapted from §3.4 of *Sketching User Experiences: The Workbook*, by Greenberg et.al.

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For each problem

(a) Sketch a picture of the situation. *Draw* the physical situation described. Then *annotate* the drawing with labels, lines, etc.

(b) Make a *note* of the information you are **given**. Try to use mathematical formulas to capture relationships between the different quantities.

(c) What is the function F that you **want** to optimize? Do you want a max or min?

(d) Find an equation for F in terms of a single variable.

(e) Find the desired absolute max/min.

1. Find two positive numbers whose sum is 25 and whose product is as small as possible.

2. Find the point on $\sqrt{x+1}$ closest to (2,0).

3. You are crossing a 2 mile wide river. Your camp is on the other side, 5 miles downstream. You are hungry, and you want to get there as quickly as possible. If you can swim at 3 mi/hr and hike at 5 mi/hr, where should you land to get there the fastest?

   You must **show all work**, including verifying that time is minimized.

4. Find two numbers x and y such that $xy^2 = 54$ and which minimizes $F = x^2 + y^2$.

5. You are building a rectangular garden. You are tired of your neighbors stealing your produce and you have looked into several types of electric fence. The material for the side of the fence near your house is \$1 per foot. The other three sides will cost \$2 per foot. If you have \$100, what is the maximum amount of area you can protect?

   You must **show all work**, including verifying that area is maximized.

6. You are designing a rectangular display box with an open top. The box should have a volume of 2 m^3, and the length of the base should equal twice the width of the base. The material for the base costs \$5 per square meter, and the material for the sides costs \$10 per square meter. Find the cost of the materials for the cheapest container.

   You must **show all work**, including verifying that this cost is a minimum.