Name: ____________________ Section: ____________________

Principles of Problem Solving [1]

1. Understand the problem
   (a) Read the problem carefully. What is *the unknown*? What are the *data*?
   (b) Draw a diagram
   (c) Introduce helpful names. For example, assign letters to the important functions of t.
2. Think of a plan
   (a) Find an equation linking the relevant parts of the problem.
   (b) Try annotating the diagram to make the connections more clear. This might mean labeling lengths of sides, looking for a triangle, etc.
3. Carry out the plan
   (a) Use implicit differentiation and the chain rule to relate the rates of change.
   (b) Use the given quantities and constraints to solve for the desired rate of change.
4. Look back
   (a) Check your work!
   (b) Can you use this result or method for another problem?

Principles of Sketching[2]

There are four basic elements of a sketch

1. **The Drawing:** Sketch the physical objects being described. Try to match the scale and relations between things.
2. **Annotations:** Add names, labels, and explanatory notes.
   - Label quantites that change over time with *letters*. If a quantity (length, angle, etc) does *not* change over time, you can label the drawing with its value.
   - You might also want to add additional lines to create a shape like a triangle, which can be used along with trigonometry or the Pythagorean theorem.
3. **Arrows:** Draw arrows to indicate motion. Once drawn, these arrows can often help you find out where to fill in the missing lines to create a triangle.
4. **Notes:** Next to your drawing, write down any formulas that may be useful for relating the relevant quantities. Common examples are area, volume, trig, simila, and distance formulas. You may also use facts about similar triangles.

[1] Adapted from Polya's *How To Solve It* and Stewart's *Calculus* 7e

[2] Adapted from §3.4 of *Sketching User Experiences: The Workbook*, by Greenberg et.al.

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For each problem

(a) What is given?

(b) What is unknown?

(c) Sketch a picture of the situation at some unknown time t.

(d) Write an equation that relates the quantities.

(e) Finish solving the problem

1. An airplane flies directly over a radar station, at a constant altitude of 3 mi above the ground. A little while later, the radar station measures that (a) the distance between the plane and radar station equals 5 mi and (b) that the distance between the plane and radar station is increasing at a rate of 500 mi/hr. What is the ground speed of the airplane at the time of the second measurement?

2. An ice cube melts, with its surface area decreasing at a rate of 3 in^2/s. How fast is the side length decreasing when the side length is 1 in?

3. A streetlight is mounted at the top of a 6 meter pole, and a 2 meter tall person is walking toward it at 2 meters per second. How fast is the length of their shadow changing when they are 4 meters from the streetlight? What about when they are 1 meter from the light?

4. A police officer is walking down a city street, when they spot a wanted felon standing 200 ft away at the corner of the next block. The police officer takes off after the felon at 12 ft/s, and the felon immediately cuts around the corner and runs away at 9 ft/s. What is the rate of change of the distance between the officer and the felon after 10 seconds have passed?

5. Suppose there is a 100 cm long water trough shaped as a triangular prism whose cross-section is an inverted triangle ∇ which is 20 cm across the top, and is 10 cm tall. If the tank is being being filled with water at a constant rate of 400 cm^3/s, how fast is the height changing when the water is half way up the tank?

6. Suppose the water trough above leaks (100 cm long, cross section is a ∇, top $=$ 20 cm, and height $=$ 1 cm). If water is being added to the tank at a rate of 400 cm^3/s, and is leaking out of the tank at 100 cm^3/s, how fast is the height changing when the water is half way up the tank?