Worksheet Problem #1

An airplane flies over a radar station...

(a) **Given:** 1. the plane is flying horizontally (altitude = 3 mi) over the station

NOTATION: let s denote the distance between plane and station at time t

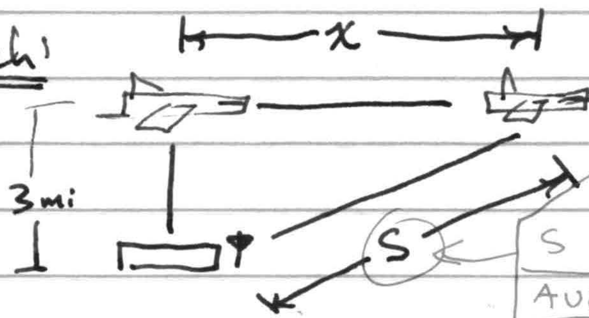
2. Know: when $s = 5$ mi, $\frac{ds}{dt} = 500$ mi/hr

(b) **Unknown:** Ground speed of the airplane.

NOTATION: let x denote the horizontal distance travelled at time t

Then $\frac{dx}{dt}$ = ground speed

(c) **Sketch:**



this distance s is changing

AVOID temptation to label this 5 mi

(d) Equation:

$$3^2 + x^2 = s^2$$

(e) finish the problem:

to relate the rates, we take $\frac{d}{dt}$ of both sides

$$\frac{d}{dt}\left(3^2 + \boxed{x}^2\right) = \frac{d}{dt}\left(\boxed{s}^2\right)$$

$$0 + 2\cdot\boxed{x}^1 \cdot \frac{d\boxed{x}}{dt} = 2\cdot\boxed{s}^1 \cdot \frac{d\boxed{s}}{dt}$$

so

$$x \cdot \frac{dx}{dt} = s \cdot \frac{ds}{dt}$$

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to find $\frac{dx}{dt}$, need: $s, x, \& \frac{ds}{dt}$. HAVE: $s \& \frac{ds}{dt}$ already

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when $s = 5$ mi, we can compute x

$$3^2 + x^2 = 5^2$$

$$9 + x^2 = 25$$

$$x^2 = 16$$

$$x = 4$$

So when $s = 5$ mi,

$$x \cdot \frac{dx}{dt} = s \cdot \frac{ds}{dt}$$

$$4 \cdot \frac{dx}{dt} = 5 \cdot (500)$$

$$\boxed{\frac{dx}{dt} = \frac{5\cdot 500}{4} = 625 \text{ mi/hr}}$$

Worksheet Problem #2

An ice cube melts, ...

(a) GIVEN: the surface area's rate of change

NOTATION: write a for the cube's surface area at time t

we are given:

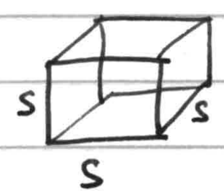
$$\frac{da}{dt} = -3 \ \frac{in^2}{s}$$

the surface area is getting smaller.

(b) Unknown: the side length's rate of change

Notation: s is the side length at time t.

(c) Sketch



(d) Note: the cube has 6 equal sides each of area s^2

$$\Rightarrow \quad a = 6 \cdot s^2$$

(e) <u>complete the problem</u>

relate $\frac{da}{dt}$ and $\frac{ds}{dt}$ by

$$\frac{d}{dt}(a) = \frac{d}{dt}\left(6 \cdot s^2\right)$$

$$\frac{da}{dt} = 2 \cdot 6 \cdot s^1 \cdot \frac{ds}{dt}$$

$$\frac{da}{dt} = 12 \cdot s \cdot \frac{ds}{dt}$$

when $s = 1''$, $\frac{da}{dt} = -3 \frac{in^3}{s}$

$$-3 = 12 \cdot 1 \cdot \frac{ds}{dt}$$

so $\boxed{\frac{ds}{dt} = \frac{-1}{4}}\ \frac{in}{sec}$

Worksheet Problem # 3

6 meter

A streetlight is mounted at the top of a pole ...

(a) Given: height of light = 6 m

a person is walking toward it

bottom of

Notation: x is the person's distance to the light pos
at time t

then

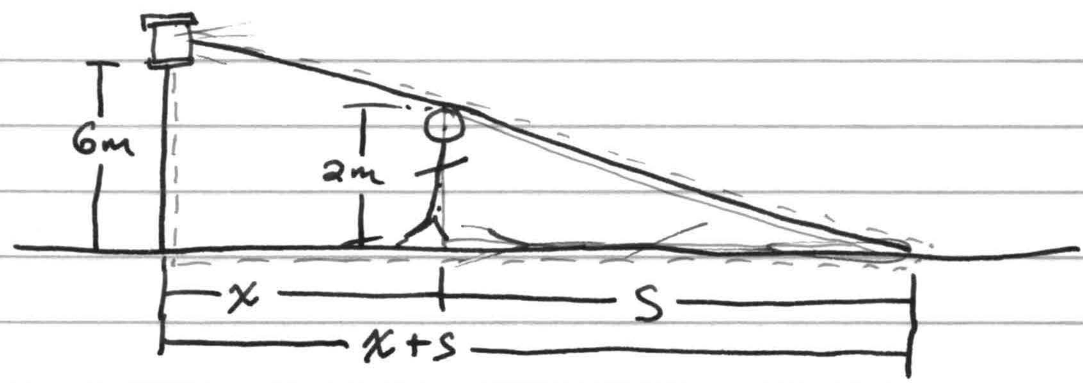
$$\frac{dx}{dt} = -2 \ \frac{m}{s}$$

because x is shrinking.

(b) Unknown: how fast their shadow's length is changing

NOTATION: S is the shadow's length
at time t

(c) sketch:



(d) equation we have 2 similar triangles

So $$\frac{S}{2} = \frac{x+S}{6}$$

(e) finish the problem:

$$\frac{s}{2} = \frac{s+x}{6}$$

So: $2s + 2x = 6s$

so $2x = 4s$

$x = 2s$

To relate the rates

$$\frac{d}{dt}\left(\boxed{x}\right) = \frac{d}{dt}\left(2\boxed{s}\right)$$

$$\frac{dx}{dt} = 2 \cdot \frac{ds}{dt}$$

Because $\frac{dx}{dt}$ is constant,

no matter how far they are from the light

$$\frac{ds}{dt} = \frac{1}{2}\frac{dx}{dt} = \frac{1}{2}(-2) = -1 \frac{m}{s}$$

Worksheet Problem #4

A police officer is walking down a street...

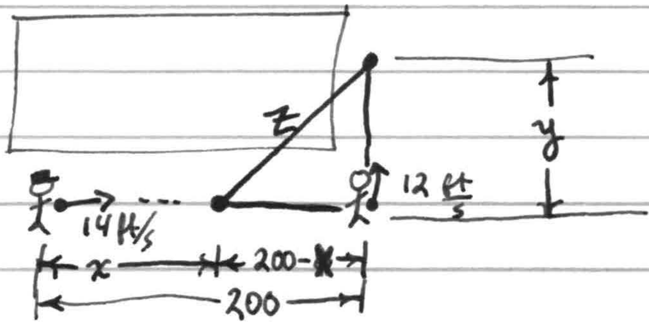
(a) Given: starting distance = 200 ft

police officer runs at 12 [redacted] $\frac{ft}{s}$

felon runs at 9 [redacted] $\frac{ft}{s}$

(b) Unknown: how fast the straight-line distance is changing

(c)



Notation: x = distance travelled by officer at time t ← know $\frac{dx}{dt}$

y = distance travelled by felon at time t ← know $\frac{dy}{dt}$

z = straight-line distance at time t ← WANT $\frac{dz}{dt}$

(e) equation

$$(200-x)^2 + y^2 = z^2$$

(f) to relate the rates

$$\frac{d}{dt}\left(\left(200-\boxed{x}\right)^2 + \left(\boxed{y}\right)^2\right) = \frac{d}{dt}\left(\boxed{z}^2\right)$$

$$\frac{d}{dt}\left[\left(\boxed{200-x}\right)^2\right] + 2\cdot y\cdot \frac{dy}{dt} = 2\cdot z\cdot \frac{dz}{dt}$$

$$2\cdot(200-x)\left(\frac{d}{dt}[200-\boxed{x}]\right) + 2y\cdot \frac{dy}{dt} = 2z\,\frac{dz}{dt}$$

$$\cancel{2}(200-x)\left(-\frac{dx}{dt}\right) + \cancel{2}y\,\frac{dy}{dt} = \cancel{2}z\,\frac{dz}{dt}$$

$$-(200-x)\frac{dx}{dt} + y\,\frac{dy}{dt} = z\,\frac{dz}{dt}$$

Notice because velocity is constant

$$x = 12t$$

and

$$y = 9t$$

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when $t = 10$ s

$x = 120$

and $y = 90$

so $z^2 = (12)^2 + (9)^2$

$z = 150$

so $-(200-120)\cdot(12) + (90)\cdot(9) = 150\cdot\frac{dz}{dt}$

solving for $\frac{dz}{dt} = \ldots = -1$ ft/sec

Worksheet Problem #5:

Suppose there is a trough of water ...

(a) Given: → dimensions & shape of trough
→ rate at which water is added

NOTATION: V = volume at time t

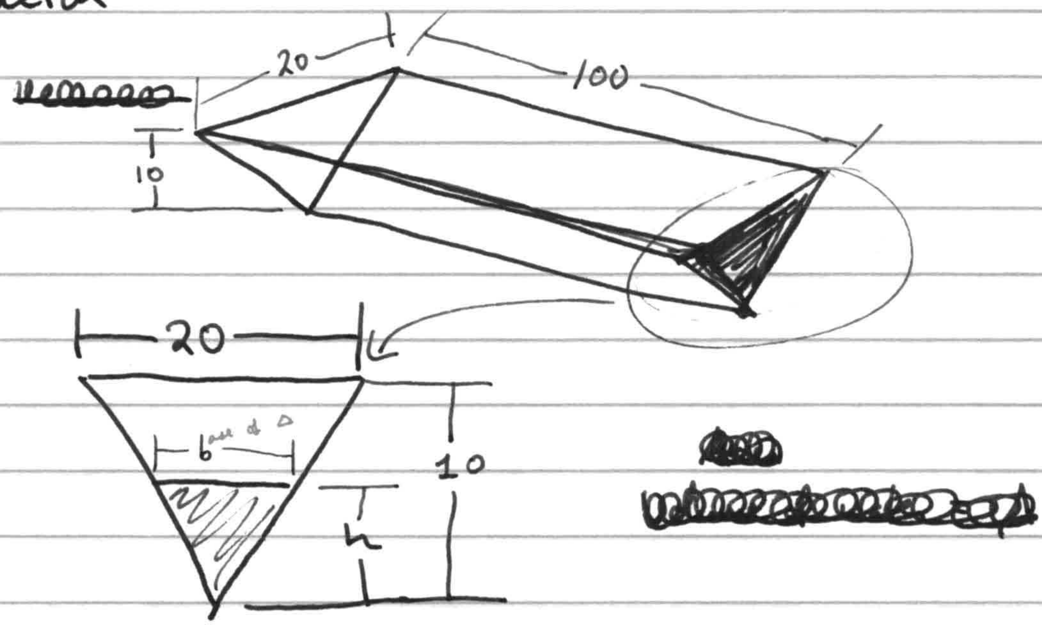
Given: $\frac{dV}{dt} = 400 \frac{cm^3}{s}$

(b) Unknown: rate of change of water height water at time t.

NOTATION: h = height at time t

Want: $\frac{dh}{dt}$.

(c) sketch



Note: ① (volume of water) = (end area)·(length)

② (end area) = $\frac{1}{2} b \cdot h$ ← we know nothing about b so we must get rid of it

and ③ $\frac{b}{h} = \frac{20}{10}$

(d) <u>equation</u>:

$$\frac{b}{h} = \frac{20}{10} \quad \Rightarrow \quad b = 2h$$

so $\quad$ end area $= \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot (2h) \cdot h = h^2$

so $\quad$ volume $= \left(\begin{smallmatrix}\text{end}\\\text{area}\end{smallmatrix}\right) \cdot (\text{length}) = 100 \cdot h^2$

to relate the rates

$$\frac{d}{dt}(V) = \frac{d}{dt}(100 \cdot h^2)$$

$$\frac{dV}{dt} = 100 \cdot 2h \cdot \frac{dh}{dt}$$

the tank is half full when $h = \frac{10}{2} = 5$.

When $\quad h = 5$

$$\frac{dV}{dt} = 200 \cdot h \cdot \frac{dh}{dt}$$

$$400 = 200 \cdot 5 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{2}{5} \frac{cm}{s}$$

Worksheet Problem #6

Suppose the water trough leaks.

Then

$$\frac{dV}{dt} = \left(\begin{matrix}\text{rate of}\\ \text{water in}\end{matrix}\right) - \left(\begin{matrix}\text{rate of}\\ \text{water out}\end{matrix}\right)$$

$$= 400 - 100$$

$$\frac{dV}{dt} = 300 \ \frac{cm^3}{s}$$

the rest of the problem is the same as it was in #5