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The Chain Rule

$$\frac{d}{dx}\Big[f\big(g(x)\big)\Big] = f'\big(g(x)\big) \cdot \frac{d}{dx}\big[g(x)\big]$$

outside $= f(u) \implies$ outside$' = f'(u)$ inside $= g(x)$

Some people like to memorize common cases of the chain rule.

If you want, you can memorize the left column as another "rule."
The right column shows that this is "rule" is an easy application of the chain rule.

Power and Chain Rule

$$\frac{d}{dx}\Big[\big(g(x)\big)^n\Big] = n\big(g(x)\big)^{n-1} \cdot \frac{d}{dx}\big[g(x)\big]$$

outside $= u^n \implies$ outside$' = n\,u^{n-1}$ inside $= g(x)$

Logarithms, Exponentials, and Chain Rule

$$\frac{d}{dx}\Big[\ln\big(g(x)\big)\Big] = \frac{1}{g(x)} \cdot \frac{d}{dx}\big[g(x)\big]$$

outside $= \ln(u) \implies$ outside$' = \frac{1}{u}$ inside $= g(x)$

$$\frac{d}{dx}\Big[e^{g(x)}\Big] = e^{g(x)} \cdot \frac{d}{dx}\big[g(x)\big]$$

outside $= e^u \implies$ outside$' = e^u$ inside $= g(x)$

Sine, Cosine, and Chain Rule

$$\frac{d}{dx}\Big[\sin\big(g(x)\big)\Big] = \cos(g(x)) \cdot \frac{d}{dx}\big[g(x)\big]$$

outside $= \sin(u) \implies$ outside$' = \cos(u)$ inside $= g(x)$

$$\frac{d}{dx}\Big[\cos\big(g(x)\big)\Big] = -\sin\big(g(x)\big) \cdot \frac{d}{dx}\big[g(x)\big]$$

outside $= \cos(u) \implies$ outside$' = -\sin(u)$ inside $= g(x)$

Notice. The shortcut rules are *quick* and *accurate*. But there isn't a conceptual difference between "doing basic chain rules in your head" and "memorizing shortcut rules."

Important. For complicated outside functions, write out the chain rule.
There are a lot of formulas, and there's no point in getting the wrong answer.

Summary. Be comfortable using the shortcuts *and* with writing out the chain rule.
There is a range of difficulty of using the chain rule. Its easy to slip up, and I doubt anyone would get $\frac{d}{dx}[\sec(x - e^{2x})]$ correct without writing everything out.

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Use Other Rules First if Needed

1. $\frac{d}{dx}\left[2\sin(3x+1) - 7e^{\cos(x)} + 5\Big(\ln(x)\Big)^3\right]$
$= 2\cdot\frac{d}{dx}[\sin(3x+1)] - 7\cdot\frac{d}{dx}[e^{\cos(x)}] + 5\frac{d}{dx}\left[(\ln(x))^3\right] = \ldots$

2. $\frac{d}{dx}\left[x \cdot e^{\cos(x)}\right] = x\cdot\frac{d}{dx}[e^{\cos(x)}] + e^{\cos(x)}\cdot\frac{d}{dx}[x] = \ldots$ $(fg)' = fg' + gf'$

3. $\frac{d}{dx}\left[\frac{\ln(3x+1)}{\ln(1-2x^2)}\right] = \frac{\ln(1-2x^2)\cdot\frac{d}{dx}[\ln(3x+1)] - \ln(3x+1)\cdot\frac{d}{dx}[1-2x^2]}{(\ln(1-2x^2))^2} = \ldots$ $\left(\frac{t}{b}\right)' = \frac{bt'-tb'}{b^2}$

Simplify First When Possible

1. $\frac{d}{dx}[x(\sqrt{x}+1)] = \frac{d}{dx}\left(x(x^{\frac{1}{2}}+1)\right) = \frac{d}{dx}\left[x^{\frac{3}{2}} + x\right] = \ldots$

2. $\frac{d}{dx}\left[\frac{x^2-4x+4}{x-2}\right] = \frac{d}{dx}\left[\frac{(x-2)\cancel{(x-2)}}{\cancel{(x-2)}}\right] = \frac{d}{dx}[x-2]$

3. $\frac{d}{dx}\left[e^{\ln(\sec(2x))}\right] = \frac{d}{dx}[\sec(2x)] = \ldots$

4. $\frac{d}{dx}\left[\frac{x^2+\sqrt{x}+1}{x}\right] = \frac{d}{dx}\left[\frac{x^2}{x} + \frac{x^{\frac{1}{2}}}{x} + \frac{1}{x}\right] = \frac{d}{dx}[x] + \frac{d}{dx}[x^{-\frac{1}{2}}] + \frac{d}{dx}[x^{-1}]$
$= \ldots$

5. $\frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right] = \frac{d}{dx}[\tan(x)]$

6. $\frac{d}{dx}\left[\ln\left(\frac{(x^2+3e^x)^{72}}{\sqrt{1-5x^2}}\right)\right] = \frac{d}{dx}\left[\ln\left((x^2+3e^x)^{72}\right) - \ln\left((1-5x^2)^{\frac{1}{2}}\right)\right]$
$= \frac{d}{dx}\left[72\cdot\ln(x^2+3e^x) - \frac{1}{2}\ln(1-5x^2)\right]$
$= \ldots$