Quiz #3 Math 1131 10/15/2014

Name: Key Section: _____

You have 12 minutes to complete the quiz. Please **show all work**, and then **circle your answer.**

1. (3 points) Compute $\frac{d}{dx}\left[\ln\left(x^2 \cdot \tan(x)\right)\right]$

(outside = $\ln(u) \Rightarrow (\text{outside})' = \frac{1}{u}$)

$= \frac{1}{x^2 \cdot \tan(x)} \cdot \frac{d}{dx}\left[x^2 \cdot \tan(x)\right]$ (1pt)

$= \frac{1}{x^2 \cdot \tan(x)} \cdot \left(x^2 \cdot \frac{d}{dx}[\tan(x)] + \tan(x) \cdot \frac{d}{dx}[x^2]\right)$

$= \frac{1}{x^2 \cdot \tan(x)} \cdot \left(x^2 \cdot \sec^2(x) + \tan(x) \cdot 2x\right)$ (2pt)

2. (3 points) Suppose that $\sin(y) + x = xy$. Compute $\frac{dy}{dx}$

① take derivative of both sides

$\frac{d}{dx}[\sin(y) + x] = \frac{d}{dx}[x \cdot y]$

$\frac{d}{dx}[\sin(y)] + \frac{d}{dx}[x] = x \cdot \frac{d}{dx}[y] + y \cdot \frac{d}{dx}[x]$

outside = sin(u), inside = y

$\cos(y) \cdot \frac{dy}{dx} + 1 = x \cdot \frac{dy}{dx} + y$ (1pt)

② collect y' terms

$\cos(y) \cdot \frac{dy}{dx} - x \cdot \frac{dy}{dx} = y - 1$

③ factor & solve

$\frac{dy}{dx}(\cos(y) - x) = y - 1 \Rightarrow \boxed{\frac{dy}{dx} = \frac{y-1}{\cos(y) - x}}$ (2pt)

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3. (4 points) Let $f(x) = x^{\sin(x)}$. Find $f'(x)$.

↑ chain

let $y = x^{\sin(x)}$

① introduce logs to both sides & break apart

$\ln(y) = \ln\left(x^{\sin(x)}\right)$

$\ln(y) = \sin(x) \cdot \ln(x)$ (1pt)

② take derivative of both sides

$\frac{d}{dx}[\ln(y)] = \frac{d}{dx}[\sin(x) \cdot \ln(x)]$

↑ chain rule ↑ product

$\frac{1}{y} \cdot y' = \sin(x) \cdot \frac{d}{dx}[\ln(x)] + \ln(x) \cdot \frac{d}{dx}[\sin(x)]$

$\frac{1}{y} y' = \sin(x) \cdot \frac{1}{x} + \ln(x) \cdot \cos(x)$ (2pt)

③ answer the question

$f'(x) = y' = y \cdot \left(\frac{\sin(x)}{x} + \ln(x) \cdot \cos(x) \right)$

$\boxed{f'(x) = x^{\sin(x)} \cdot \left(\frac{\sin(x)}{x} + \ln(x) \cdot \cos(x) \right)}$ (1pt)