6:33

Instructions:

- This exam contains 16 pages. When we begin, check you have *one* of each page.
- You will have 2 hours to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.** In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:

By writing my name below, I agree that all the work which appears on this exam is entirely my own.

I will not look at other peoples' work, and I will not communicate with anyone else about the exam.

I will not use any calculators, notes, etc.

I understand that violating the above carries *serious consequences*, both moral and academic.

Printed Name: Key Signature: ____________________

Section: ____________________

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Points:	12	12	10	10	12	12	10	10	12	12	10	4	6	6	12	150
Score:																

1. (a) [4 points] Let $f(x) = \sqrt{2x+1}$. Find $f'(x)$.

$$f(x) = (2x+1)^{\frac{1}{2}}$$

$$f'(x) = \cancel{\tfrac{1}{2}} \cdot (2x+1)^{-\frac{1}{2}} \cdot \cancel{2}$$

$$= \frac{1}{\sqrt{2x+1}}$$

(b) [4 points] Let $f(x) = \dfrac{x}{\sin(x)}$. Find $f'(x)$.

$$\boxed{\frac{b \cdot t' - t \cdot b'}{b^2}}$$

$$f'(x) = \frac{d}{dx}\left[\frac{x}{\sin(x)}\right] = \frac{\sin(x) \cdot 1 - x \cdot \cos(x)}{\sin^2(x)}$$

(c) [4 points] Let $y = \ln(2x)$. Find $\dfrac{d^2y}{dx^2}$.

$$\frac{dy}{dx} = \frac{1}{2x} \cdot 2 = \frac{1}{x}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{x^2}$$

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2. [12 points] Compute the following limits, showing your work.

(a) Compute the limit $\lim_{x\to 3^+} \frac{x^2+4x+3}{x^2-9}$ $= \lim_{x\to 3^+} \frac{(\cancel{x+3})(x+1)}{(\cancel{x+3})(x-3)}$

2pt $= \lim_{x\to 3^+} \frac{(x+1)}{(x-3)}$

think: $\approx \frac{4}{\text{small pos}}$

$\approx$ big pos

2pt $= \infty$

(b) Compute the limit $\lim_{x\to 1^-} \frac{x^2-4}{x^2-25}$ $= \lim_{x\to 1^-} \frac{(x-2)(x+2)}{(x+5)(x-5)}$

2pt Think $\approx \frac{(1-2)(1+2)}{(1+5)(1-5)}$

$\approx \frac{-3}{6\cdot(-4)} = \frac{-3}{-24}$

2pt $= \frac{1}{8}$

W31

Correct Sgz thm:

$|\,|\tfrac{1}{2}\,|\,|\,|\,|$

Correct answer:

$|\,|\,|$

(c) Compute the limit $\lim_{x\to\infty} \frac{\cos(x)}{e^x}$

2pt think: $\approx \frac{\#\text{ betw } -1 \,\&\, 1}{\text{HUGE}}$

$\approx$ tiny

2pt $= 0.$

6138

3. [10 points] Suppose that a population of bacteria is growing in a petri dish. Suppose also that the first time you look at the dish you count 50 bacteria, and that you count 300 bacteria in the dish 2 hours later.

150

(a) Find a formula for the population as a function of the number of hours t since your first measurement.

$$P(t) = 50 \cdot e^{kt}$$

$$p(2) = \cancel{300}\ 150 = 50e^{k \cdot 2}$$

$$\cancel{6}\ 3 = \frac{\cancel{300}\ 150}{50} = e^{k \cdot 2}$$

$$\ln(3) = k \cdot 2$$

$$k = \frac{\ln(3)}{2}$$

$$P(t) = 50 \cdot e^{\left(\frac{\ln(3)}{2} t\right)}$$

(b) How much time is required for the population to expand to 5000?

find t s.t.

$$5000 = 50 \cdot e^{\left(\frac{\ln(3)}{2} t\right)}$$

$$100 = e^{\frac{\ln(3)}{2} t}$$

$$\ln(100) = \frac{\ln(3)}{2} t$$

$$t = \frac{2 \cdot \ln(100)}{\ln(3)}$$

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4. (a) [5 points] Use L'Hopital's rule to compute $\lim_{x\to\infty} \frac{e^x}{x^2}$ $\to\infty$ $\to\infty$

2pt $\left\{ \overset{H}{=} \lim_{x\to\infty} \frac{e^x}{2x} \right.$ $\to\infty$ $\to\infty$

2pt $\left\{ \overset{H}{=} \lim_{x\to\infty} \frac{e^x}{2} \right.$

1pt $\left\{ = \infty \right.$

(b) [5 points] Use L'Hopital's rule to compute the limit $\lim_{x\to 1^+} \frac{\ln(x)}{x-1}$ $\to 0$ $\to 0$

3pt $\left\{ \overset{H}{=} \lim_{x\to 1} \frac{\frac{1}{x}}{1} \right.$

2pt $\left\{ = 1 \right.$

 6:42

5. [12 points] Let $f(x) = e^{-2x} + x$.

(a) Find the linearization of $f(x)$ at $a = 0$.

$$L_0(x) = f'(0)(x-0) + f(0)$$

2pt

$$f'(x) = \frac{d}{dx}\left[e^{-2x} + x\right] = e^{-2x}\cdot(-2) + 1$$

2pt

$$f'(0) = e^{0}\cdot(-2) + 1 = -1$$

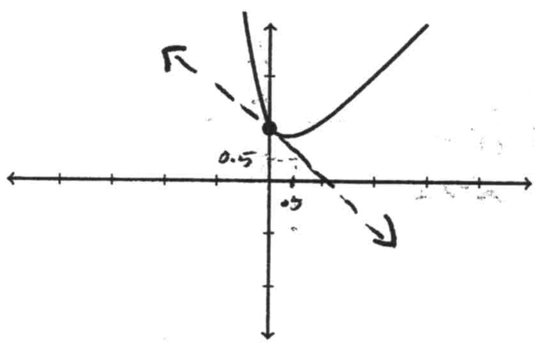
$$f(0) = e^{0} + 0 = 1$$

2pt

$$L_0(x) = -1(x-0) + 1 = -x + 1$$

(b) Sketch the line tangent to the curve at the point $(0, 1)$.

2pt



(c) Use the linearization $L_0(x)$ of $f(x)$ at $a = 0$ to approximate $f(0.5)$. Does this provide a good or a bad approximation?

4pt

$$f(0.5) \approx L_0(0.5) = -(0.5) + 1$$

$$= 0.5$$

not that good an approx... see picture

6:44

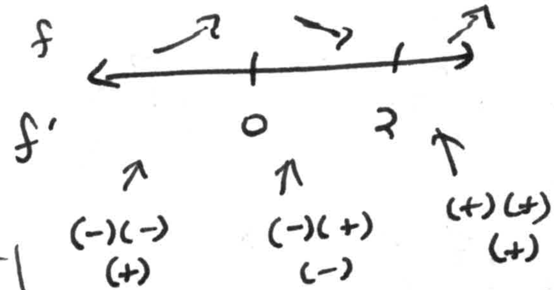
6. [12 points] Let $f(x) = x^3 - 3x^2 + 1$

Find the following if they exist (or write DNE). You must **show all work**.

1. Find the intervals where $f(x)$ is increasing/decreasing. Identify which is which.

$f'(x) = 3x^2 - 6x$

$= 3x(x-2)$

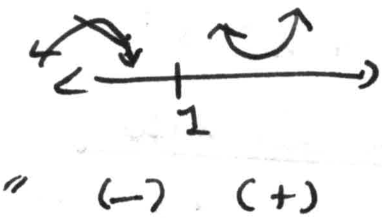


increasing on $(-\infty, 0) \cup (2, \infty)$

decreasing on $(0, 2)$

2. Find the intervals where $f(x)$ is concave up/down. Identify which is which.

$f''(x) = 6x - 6 = 6(x-1)$



concave down on $(-\infty, 1)$

concave up on $(1, \infty)$

3. Find the x value(s) of the local maxima and local minima of f. Identify which is which.

local max at 0

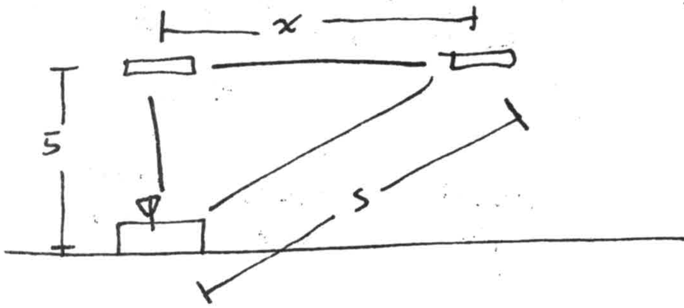
local min at 2

4. Find the x value(s) of the inflection points of f.

inflection pt at 1.

6:46

7. [10 points] An airplane flies directly over a radar station, at a constant altitude of 5 mi above the ground. A little while later, the radar station measures that (a) the distance between the plane and radar station equals 10 mi and (b) that the distance between the plane and radar station is increasing at a rate of 250 mi/hr. What is the ground speed of the airplane at the time of the second measurement?



Dynamic Sketch (3pt)

① Relate the functions

$$5^2 + x^2 = s^2$$

$$25 + x^2 = s^2$$

② Relate the rates

$$\frac{d}{dt}\left[25 + x^2\right] = \frac{d}{dt}\left[s^2\right]$$

$$2 \cdot x \cdot \frac{dx}{dt} = 2 \cdot s \cdot \frac{ds}{dt}$$

Relate the fns (3pt)

③ answer the question

when $s = 10$, $\frac{ds}{dt} = 250$

& $5^2 + x^2 = 10^2$

$25 + x^2 = 100$

$x^2 = 75$

$x = \sqrt{75}$

(2pt)

so

$$2 \cdot (\sqrt{75}) \cdot \frac{dx}{dt} = 2 \cdot 10 \cdot 250$$

$$\frac{dx}{dt} = \frac{5000}{2 \cdot (\sqrt{75})} \text{ mi/hr.} = \frac{2500}{\sqrt{75}}$$

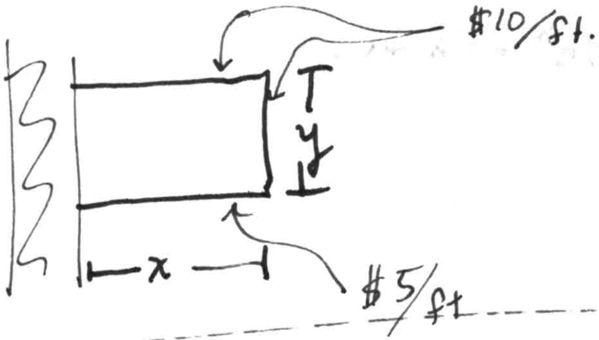
(2pt)

6:49

8. [10 points] You decide to build walls for your wilderness camp. A deep ravine will form the west boundary. The material for the southern wall will cost \$5 per foot, and the material for the north and east walls will cost \$10 per foot.

If you have \$100, what is the area of the largest camp you can protect?

You must **show all work**, including verifying that area is maximized.



cost = $5x + 10y + 10x$

$= 15x + 10y$ (2pt)

maximize area subject to Constraint

cost = 100

$100 = 15x + 10y$

$10y = 100 - 15x$

$y = 10 - \frac{15}{10}x$

$y = 10 - \frac{3}{2}x$ (2pt)

$F = xy$

$F = x\left(10 - \frac{3}{2}x\right)$

$F = 10x - \frac{3}{2}x^2$ (2pt)

$F'(x) = 10 - 3x$

F'(x) always exists

F'(x) = 0 when

$0 = 10 - 3x$

$10 = 3x$

$x = \frac{10}{3}$

(2pt)

F

F′ (+) $\frac{10}{3}$ (−)

Max area when $x = \frac{10}{3}$

max area is

$F\left(\frac{10}{3}\right) = 10 \cdot \frac{10}{3} - \frac{3}{2} \cdot \left(\frac{10}{3}\right)^2.$

(2pt)

6:53

9. [12 points] (a) Compute the general antiderivative for $f(x) = 42x + 9e^x + 5$.

$$F(x) = \frac{42x^2}{2} + 9 \cdot e^x + 5x + C$$

$$= 21x^2 + 9e^x + 5x + c.$$

(b) Suppose that $f''(x) = x^2 + 12x + \cos(x)$, that $f'(0) = 2$ and that $f(0) = 4$. Find a formula for $f(x)$.

$$f'(x) = \frac{x^3}{3} + \frac{12x^2}{2} + \sin(x) + c$$

$$\boxed{\begin{array}{l} f'(0) = 2 = 0 + 0 + \underbrace{\sin(0)}_{=0} + c \\ c = 2 \end{array}}$$

$$f'(x) = \frac{x^3}{3} + 6x^2 + \sin(x) + 2$$

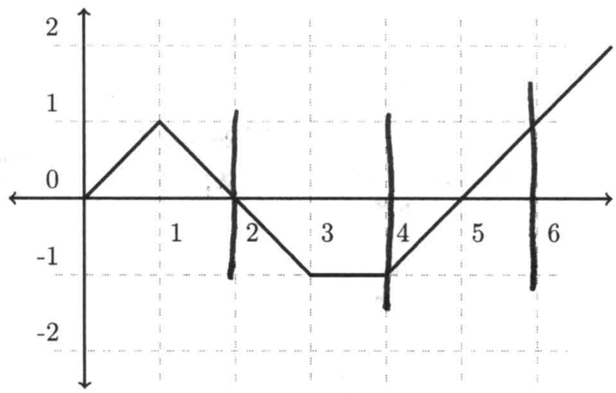
$$f(x) = \frac{x^4}{3 \cdot 4} + \frac{6x^3}{3} + (-\cos(x)) + 2x + D$$

$$\boxed{\begin{array}{l} f(0) = 4 = 0 + 0 + (-1) + 0 + D \\ D = 5 \end{array}}$$

$$\boxed{= \frac{x^4}{12} + 2x^3 - \cos(x) + 2x + D}$$

6:55

10. [12 points] Suppose that the function $f(x)$ is given by the following graph.



Let $A(x) = \int_0^x f(t)\,dt$. Compute the following

(a) $A(2)$ $= \frac{1}{2} + \frac{1}{2} = 1$

(b) $A(4)$ $= \frac{1}{2} + \frac{1}{2} - \frac{1}{2} - 1 = \frac{-1}{2}$

(c) $A(6) = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} - 1 - \frac{1}{2} + \frac{1}{2} = \frac{-1}{2}$

(d) $A'(2)$ $= f(2) = 0$

(e) $A'(4)$ $= f(4) = -1$

(f) $A'(6)$ $= f(6) = 1.$

$A'(x) = \frac{d}{dx}\left[\int_0^x f(t)\,dt\right]$

$= f(x)$

6:57.

11. [10 points]

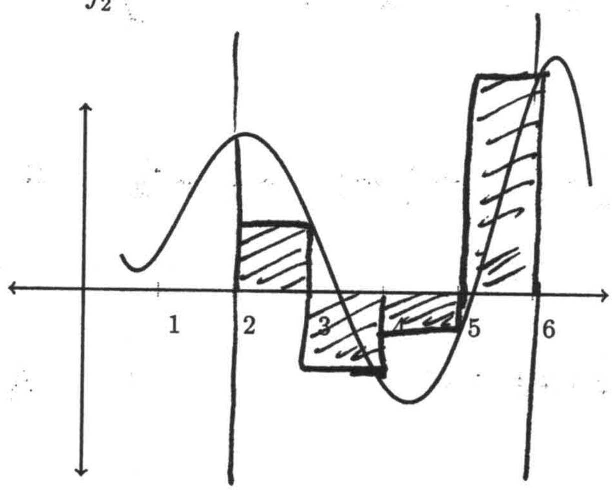
(a) Write down the definite integral expressed by the following limit of Reimann sums

$$\underbrace{\lim_{n\to\infty}\Big[\sum_{i=1}^{n}}_{\int_a^b}\Big(\cos\Big(\underbrace{2+i\cdot\frac{6-2}{n}}_{x_i}\Big)+5\Big)\cdot\underbrace{\frac{6-2}{n}}_{\Delta x}\Big]$$

(2pts) - limits of integration

$$= \int_2^6 [\cos(x) + 5]\,dx$$ (2pts) - function

(b) Let $f(x)$ be defined using the following graph. Sketch and shade in the rectangles you would use to approximate $\int_2^6 f(x)\,dx$ using right sums and 4 rectangles.

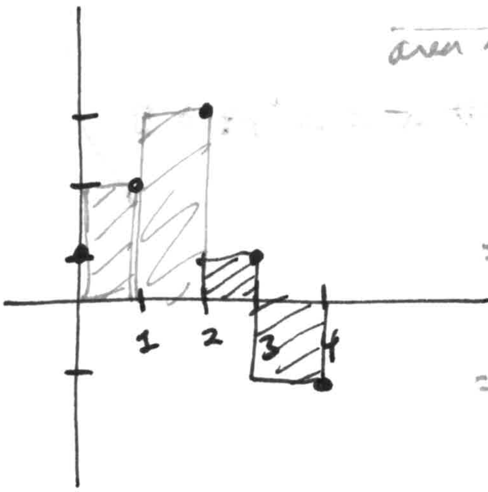


(6pts)

12. [4 points] Let $f(x)$ be a function which takes on the values in the following table.

x	0	1	2	3	4
$f(x)$	1	2	3	1	-1

Use right sums and ~~2~~ 4 rectangles to approximate the area under $f(x)$ between ~~2~~ 0 and 4.



area $\approx f(1)\cdot\Delta x + f(2)\cdot\Delta x$

$+ f(3)\cdot\Delta x + f(4)\cdot\Delta x$

$= 2\cdot\Delta x + 3\Delta x + 1\cdot\Delta x + -1\cdot\Delta x$

$= 2\cdot 1 + 3\cdot 1 \quad 1\cdot 1 + -1\cdot 1$

$= 5 \qquad = 0.$

7:01

13. [6 points] Compute the following integral.

Compute $\int_1^2 \boxed{\frac{x}{x^2+4}\,dx}$ $\frac{du}{2}$

u

(4pt)

$u = x^2 + 4$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$x = 1 \Rightarrow u = 1^2 + 4 = 5$

$x = 2 \Rightarrow u = 2^2 + 4 = 8$

$= \int_{u=5}^{u=8} \frac{1}{u} \cdot \frac{du}{2}$

(2pt)

$= \frac{\ln|8|}{2} - \frac{\ln|5|}{2}$

14. [6 points] Compute the following integral.

Compute $\displaystyle\int_2^3 \frac{x^2+4}{x}\,dx$

(2pt)

$$= \int_2^3 \frac{x^2}{x} + \frac{4}{x}\,dx$$

$$= \int_2^3 x + \frac{4}{x}\,dx$$

(2pt)

$$= \left.\frac{x^2}{2} + 4\cdot\ln|x|\,\right]_2^3$$

(2pt)

$$= \left(\frac{3^2}{2} + 4\cdot\ln|3|\right) - \left(\frac{2^2}{2} + 4\cdot\ln|2|\right)$$

$$= \frac{9}{2} + 4\cdot\ln(3) - \underset{\frac{4}{2}}{2} - 4\cdot\ln(2)$$

$$= \frac{5}{2} + 4\cdot\ln\left(\frac{3}{2}\right)$$

7:03.

15. [12 points] Compute the following integrals.

(a) Compute $\int \frac{\sin\left(\ln(x)\right)}{\boxed{x}}\boxed{dx}$

(2pt) $u = \ln(x)$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}\,dx$

(2pt) $= \int \sin(u)\cdot \frac{du}{\cancel{2}}$

$= \frac{-\cos(u)}{\cancel{2}} + C$

(2pt) $= \frac{-\cos(\ln(x))}{\cancel{2}} + C$

(b) Compute $\int \left[x \cdot e^{(x^2)} + 2x\right]\,dx$

$= \int x \cdot e^{(x^2)}\,dx + \int 2x\,dx$

(2pt) $u = x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$= \int e^u \cdot \frac{du}{2} + \frac{2x^2}{2} + C$

(2pt) $= \frac{e^u}{2} + \underbrace{x^2 + C}_{(2pt)}$

$\boxed{= \frac{e^{x^2}}{2} + x^2 + C}$

7:05.