Instructions:

- This exam contains 11 pages. When we begin, check you have *one* of each page.
- You will have 70 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.**
  In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:

By writing my name below, I agree that all the work
which appears on this exam is entirely my own.

I will not look at other peoples' work,
and I will not communicate with anyone else about the exam.

I will not use any calculators, notes, etc.

I understand that violating the above carries *serious consequences*,
both moral and academic.

Printed Name: Key Signature: ______

Section: ______

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	10	10	10	6	10	12	10	10	12	100
Score:											

1. (a) [5 points] Let $f(x) = \sin(\cos(2x))$. Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[\sin\left(\cos(2x)\right)\right]$$

2pt $$= \cos(\cos(2x)) \cdot \frac{d}{dx}\left[\cos(2x)\right]$$

2pt $$= \cos(\cos(2x)) \cdot (-\sin(2x)) \cdot \frac{d}{dx}[2x]$$

1pt $$= -2 \cdot \cos(\cos(2x)) \cdot \sin(2x)$$

(a) ____________

(b) [5 points] Let $f(x) = x^2 \cdot \ln(x)$. Find $f''(x)$.

$$f'(x) = 2x \cdot \ln(x) + x^2 \cdot \frac{1}{x}$$

2pt $$= 2x \cdot \ln(x) + x$$

2pt $$f''(x) = 2 \cdot \ln(x) + 2x \cdot \frac{1}{x} + 1$$

1pt $$\boxed{f''(x) = 2 \cdot \ln(x) + 3}$$

(b) ____________

2. (a) [5 points] Let $x = y \cdot \sin(x) + y^2$. Find $\frac{dy}{dx}$.

$$\frac{d}{dx}[x] = \frac{d}{dx}\left[y \cdot \sin(x)\right] + \frac{d}{dx}\left[y^2\right]$$

2pt $$1 = y \cdot \cos(x) + y' \cdot \sin(x) + 2 \cdot y \cdot y'$$

2pt $$1 - y \cdot \cos(x) = y' \left(\sin(x) + 2y\right)$$

1pt $$\boxed{y' = \frac{1 - y \cdot \cos(x)}{\sin(x) + 2y}}$$

(b) [5 points] Let $f(x) = \Big(3x + 1\Big)^{2x}$. Find $f'(x)$.

$$y = (3x+1)^{2x}$$

2pt $$\frac{d}{dx}\left[\ln(y)\right] = \frac{d}{dx}\left[\ln\left((3x+1)^{2x}\right)\right] = \frac{d}{dx}\left[2x \cdot \ln(3x+1)\right]$$

2pt $$\frac{1}{y} \cdot y' = 2 \cdot \ln(3x+1) + 2x \cdot \frac{1}{3x+1} \cdot 3$$

$$y \cdot \frac{1}{y} \cdot y' = y \cdot \left(2 \cdot \ln(3x+1) + \frac{6x}{3x+1}\right)$$

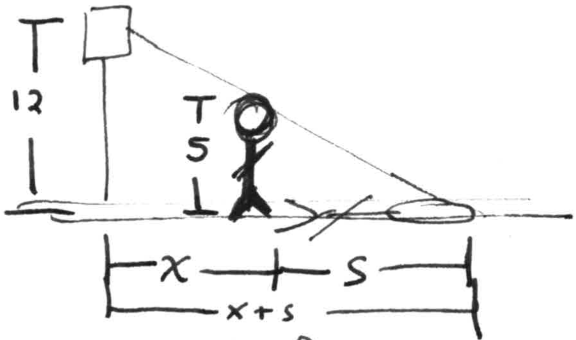
1pt $$f'(x) = y' = (3x+1)^{2x} \cdot \left(2 \cdot \ln(3x+1) + \frac{6x}{3x+1}\right)$$

7:18

3. [10 points] A streetlight is mounted at the top of a 12 foot pole, and a 5 foot tall person is walking toward it at 3 feet per second. $\frac{dx}{dt} = -3$

How fast is the length of their shadow changing when they are 3 foot away from the streetlight?

what is $\frac{ds}{dt}$ when $x = 3$?



3pt dynamic sketch

① Relate the functions

$$\frac{s}{5} = \frac{x+s}{12}$$

4pt relate fns

$$12s = 5(x+s) = 5x + 5s$$

$$7s = 5x$$

② Relate the Rates

$$\frac{d}{dt}[7s] = \frac{d}{dt}[5x]$$

2pt relate rates

$$7 \cdot \frac{ds}{dt} = 5 \cdot \frac{dx}{dt}$$

③ Answer the Question

$$7 \cdot \frac{ds}{dt} = 5(-3)$$

$$\frac{ds}{dt} = \frac{-15}{7} \text{ in/s}$$

1 pt answer qu.

4. (a) [5 points] Use L'Hospital's rule to compute the following limit

$$\lim_{x\to\infty} \frac{x^2 \to \infty}{\ln(x) \to \infty}$$

You must **show all work** for credit.

3pt $\overset{H}{=} \lim_{x\to\infty} \dfrac{2x}{\frac{1}{x}} \cdot \dfrac{x}{x}$

2pt $= \lim_{x\to\infty} \dfrac{2x^2}{1}$

$= \infty$

(b) [5 points] Use L'Hospital's rule to compute the following limit

$$\lim_{x\to 0^+} \underset{\downarrow\, 0}{x} \cdot \underset{\downarrow\, -\infty}{\ln(x)}$$

You must **show all work** for credit.

2pt $= \lim_{x\to 0^+} \dfrac{\ln(x) \to -\infty}{\frac{1}{x} \to \infty}$

2pt $\overset{H}{=} \lim_{x\to 0^+} \dfrac{\frac{1}{x}}{\frac{-1}{x^2}} \cdot \dfrac{x^2}{x^2}$

1pt $= \lim_{x\to 0^+} \dfrac{x}{-1} = 0$

5. [6 points] Use L'Hospital's rule to compute the following limit

$$\lim_{x\to 0^+} \left(1+2x\right)^{1/x}$$

You must **show all work** for credit.

$$y = (1+2x)^{1/x}$$

$$\ln(y) = \ln\left((1+2x)^{1/x}\right) = \frac{1}{x}\cdot \ln(1+2x)$$

$$\lim_{x\to 0^+} \boxed{\ln(y)} = \lim_{x\to 0^+} \frac{\ln((1+2x))}{x} \begin{matrix}\to 0\\ \to 0\end{matrix}$$

2pt

$$\overset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{1+2x}\cdot 2}{1}$$

2pt

$$= \boxed{2}$$

$$= \lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\boxed{\ln(y)}}$$

2pt

$$= e^2$$

7:2■5

6. (a) [5 points] Compute the linearization of $f(x) = \sin(x) + \cos(x)$ at $a = 0$.

$$L_0(x) = f'(0)\cdot(x-0) + f(0)$$

2pt

$$f'(x) = \cos(x) - \sin(x)$$

$$f'(0) = \cos(0) - \sin(0) = 1 - 0 = 1$$

$$f(0) = \sin(0) + \cos(0) = 0 + 1 = 1$$

Max 2pt/5pt if $L_0(x)$ is not a LINE

3pt

$$L_0(x) = 1(x-0) + 1 = x+1$$

(b) [5 points] Use linear approximations to estimate $f(0.5)$ You must simply completely, and give your answer as a **decimal**.

$$f(0.5) \approx L_0(0.5) = 0.5 + 1 = 1.5$$

7126

7. [12 points] Let $f(x) = \dfrac{x^4}{4} - x^3 + 2$.

Find the following. If a requested quantity doesn't exist, answer DNE.

(a) The intervals where $f(x)$ is increasing/decreasing. Identify which is which.

$$f'(x) = \frac{4x^3}{4} - 3x^2 + 0 = x^3 - 3x^2 = x^2(x - 3)$$

1pt

f: ↘ 0 ↘ 3 ↗

f': $(-)^2(-)$, $(+)(-)$, $(-)$ | $(+)^2(-)$, $(-)$ | $(+)^2(+)$, $(+)$

increasing on $(3, \infty)$

Decreasing on $(-\infty, 0) \cup (0, 3)$

(b) The intervals where $f(x)$ is concave up/down. Identify which is which.

$$f''(x) = 3x^2 - 6x = 3x(x - 2)$$

1pt

f: ∪ 0 ∩ 2 ∪

f'': $(-)(-)$, $(+)$ | $(+)(-)$, $(-)$ | $(+)(+)$, $(+)$

concave up on $(-\infty, 0) \cup (2, \infty)$

concave down on $(0, 2)$

1pt en for intervals x-vals

(c) The x value(s) of the local maxima and local minima of f.

~~No local max~~ Local max DNE

local min at $x = 3$

(d) The x value(s) of the inflection points of f.

inflection pts [redacted] at $x = 0$ and $x = 2$

8. [10 points] Find two numbers x and y whose product is as large as possible such that $2x+y=3$.

You must **show all work** for credit, including verifying that your answer is optimal.

Maximize product subject to constraint

$F = xy$

$2x+y=3$

$y = 3-2x$

$F = x(3-2x)$

4pt $F(x) = 3x - 2x^2$

$F'(x) = 3 - 4x$

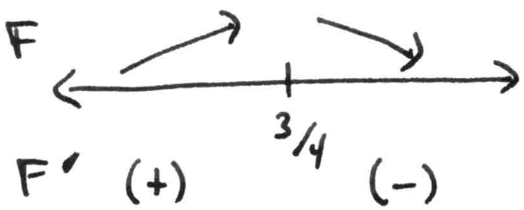
2pt

$F'(x)=0$ when $3=4x$

$x = 3/4$

$F'(x)$ always exists

2pt



2pt

product is maximized when

$x = 3/4$

and

$y = 3 - 2(3/4) = 3 - 3/2$

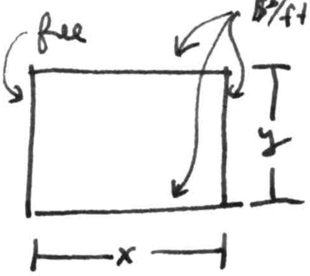
$= 3/2$

7:32

9. [10 points] For tax reasons, you are building a 40 ft^2 rectangular garden. Your house will form the west boundary of the yard, the fence for the other three sides will cost \$3 per foot. What are the dimensions of the garden with the least cost?

You must **show all work** for credit, including verifying that your answer is optimal.

~~Minimize cost~~



Minimize cost subject to constraint

$F = 3x + 3y + 3x$ 3pt

$F(x) = 6x + 3\left(\frac{40}{x}\right)$

$F(x) = 6x + \frac{120}{x}$ 2pt

area $= 40 = x \cdot y$

$y = \frac{40}{x}$

$F'(x) = 6 + 120 \cdot \frac{-1}{x^2}$

2pt

$F'(x) = 0$ when

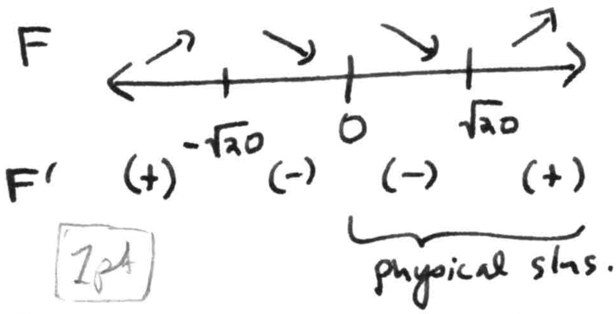
$6 = \frac{120}{x^2}$

$6x^2 = 120$

$x^2 = 20$

$x = \pm\sqrt{20}$

$F'(x)$ DNE when $x = 0$



1pt

Cost minimized when

2pt $x = \sqrt{20}$

and $y = \frac{40}{\sqrt{20}}$

also acceptable $x = 2\sqrt{5}$ $= \frac{40}{\sqrt{80}}$

also acceptable: $y = \frac{40\sqrt{20}}{20} = 2\sqrt{20} = 4 \cdot \sqrt{5} = \sqrt{80}$

7:36

10. [12 points] (a) Compute the general antiderivative of $f(x) = \dfrac{3x^2+2}{x^2}$

1pt

$$f(x) = 3 + 2x^{-2}$$

$$F(x) = 3x + 2\cdot\frac{x^{-2+1}}{-2+1} + C = 3x + 2\cdot(x^{-1})\cdot(-1) + C$$

$$F(x) = \underbrace{3x - \frac{2}{x}}_{2pt} + C$$

1 pt

(b) Suppose that $f''(x) = e^x + \cos(x)$, that $f(0) = 2$ and that $f'(0) = 3$. Find $f(x)$.

$$f'(x) = e^x + \sin(x) + C$$

2pt

$$f'(0) = 3 = e^0 + \sin(0) + C$$

$$3 = 1 + \sin(0) + C$$

$$C = 2$$

2pt

$$f'(x) = e^x + \sin(x) + 2$$

$$f(x) = e^x + (-\cos(x)) + 2x + D$$

2pt

$$f(0) = 2 = e^0 - \cos(0) + 2\cdot 0 + D$$

$$2 = 1 - 1 + D$$

$$D = 2$$

2pt

$$\boxed{f(x) = e^x - \cos(x) + 2x + 2}$$