Instructions:

- This exam contains 10 pages. When we begin, check you have *one* of each page.
- You will have 70 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.** In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:
By writing my name below, I agree that:
All the work which appears on this exam is entirely my own.
I will not look at other peoples' work,
and I will not communicate with anyone else during the exam.
I will not use any calculators, notes, etc.
I understand that violating the above carries *serious consequences*,
both moral and academic.

Printed Name: Key Signature: ____________

Section: ____________

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	15	8	8	8	12	15	10	14	100
Score:										

exp growth

1. [10 points] Suppose that a mysterious zombie outbreak begins in Waterbury, and grows at a rate proportional to its size. Suppose that the outbreak starts out with 10 zombies, and that you observe 1000 zombies in waterbury after 3 days have passed.

   (a) Find an equation $P(t)$ for the population as a function of t in days.

2 $$P(t) = P_0 \cdot e^{kt} = 10 \cdot e^{kt}$$

2 $$P(3) = 1000 = 10 \cdot e^{k \cdot 3}$$

$$100 = e^{k \cdot 3}$$

$$\ln(100) = k \cdot 3$$

$$k = \frac{\ln(100)}{3}$$

$$P(t) = 10 \cdot e^{\frac{\ln(100)}{3} t}$$

1

   (b) How long must you wait until there are 2000 zombies?

Want t s.t.

2 $$2000 = 10 \cdot e^{\left[\frac{\ln(100)}{3} t\right]}$$

$$200 = e^{\frac{\ln(100)}{3} t}$$

$$\ln(200) = \frac{\ln(100)}{3} t$$

3 $$t = \frac{3 \cdot \ln(200)}{\ln(100)}$$

9:22

2. Evaluate the following limits. Be sure to **show your work.**

(a) [5 points] $\lim\limits_{x\to 2^-} \dfrac{x^2-2}{x^2+4}$ ← continuous at 2 $\Rightarrow$ can plug in 2

4pt

$$= \frac{2^2-2}{2^2+4}$$

$$= \frac{2}{8} = \frac{1}{4}$$

1pt

(a) $\frac{1}{4}$

(b) [5 points] $\lim\limits_{x\to 5^-} \dfrac{x^2-3x-10}{x^2-25}$ ← cannot plug in 2

$$= \lim_{x\to 5^-} \frac{\cancel{(x-5)}(x+2)}{\cancel{(x-5)}(x+5)}$$

3pt

$$= \lim_{x\to 5^-} \frac{(x+2)}{(x+5)}$$ ← can plug in 2

$$= \frac{5+2}{5+5}$$

(b) $\frac{7}{10}$

2pt

(c) [5 points] $\lim\limits_{x\to 1^-} \dfrac{x^2+5x+4}{x^2+3x-4}$ ← cannot plug in 1

$$= \lim_{x\to 1^-} \frac{\cancel{(x+4)}(x+1)}{\cancel{(x+4)}(x-1)}$$

2pt

$$= \lim_{x\to 1^-} \left(\frac{x+1}{x-1}\right)$$ ← cannot plug in 1

think

$\approx \dfrac{(\text{just under } 1 + 1)}{(\text{just under } 1 - 1)}$

$\approx \dfrac{\text{just under } 2}{\text{small neg}}$

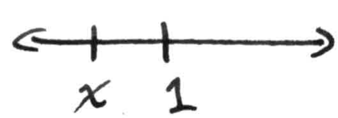
$\approx$ Big neg

2pt

$= -\infty$

1pt

(c) $-\infty$



3. Evaluate the following limits. ***Remember*** you must **show work**.

(a) [4 points] $\displaystyle\lim_{x\to-\infty} \frac{5x - 4x^2 + 3}{2x^2 - 4x + 6}$

fastest growing

$$= \lim_{x\to-\infty} \frac{\cancel{x^2}}{\cancel{x^2}} \cdot \frac{\left(\frac{5}{x} - 4 + \frac{3}{x^2}\right)}{\left(2 - \frac{4}{x} + \frac{6}{x^2}\right)}$$

$\frac{5}{x} \to 0$, $\frac{3}{x^2} \to 0$, $\frac{4}{x} \to 0$, $\frac{6}{x^2} \to 0$

3pt

$$= \frac{-4}{2}$$

(a) -2

2pt

(b) [4 points] $\displaystyle\lim_{x\to\infty} \frac{27 - x^3}{x^2 + 2}$

$$= \lim_{x\to\infty} \frac{x^3}{x^2} \cdot \frac{\left(\frac{27}{x^3} - 1\right)}{\left(1 - \frac{2}{x^2}\right)}$$

$\frac{27}{x^3} \to 0$, $\frac{2}{x^2} \to 0$

3pt

$$= \lim_{x\to\infty} x \cdot \left(\frac{-1}{1}\right)$$

think
$\approx$ ([illegible])(us debt)(-1)
$\approx$ BIG NEG

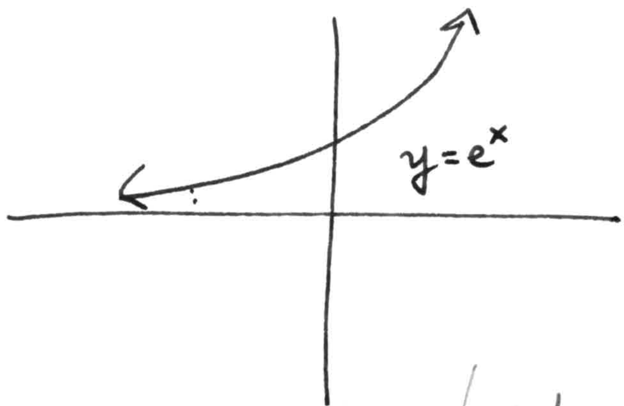
(b)

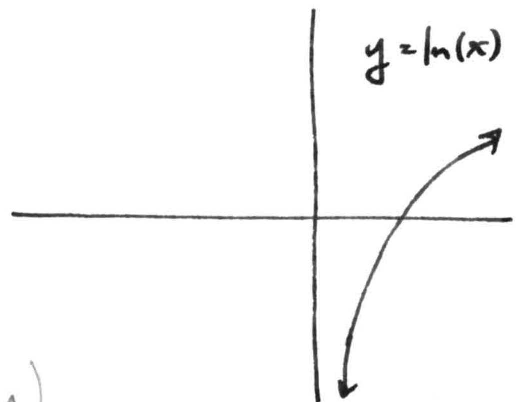
$$= -\infty$$

2pt

9:40

4. (a) [4 points] Sketch the graphs of $f(x) = e^x$ and $f(x) = \ln(x)$.





(2pt each)

(b) [4 points] Compute $\lim_{x \to 1^-} e^{1/(x-1)}$. *Remember* to **show work.**

cannot plug in 1.

think

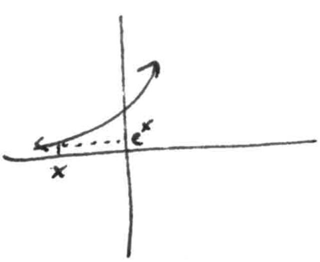
$\approx e^{1/(\text{just under } 1 - 1)}$

$\approx e^{1/(\text{small neg})}$

$\approx e^{\text{BIG NEG}}$

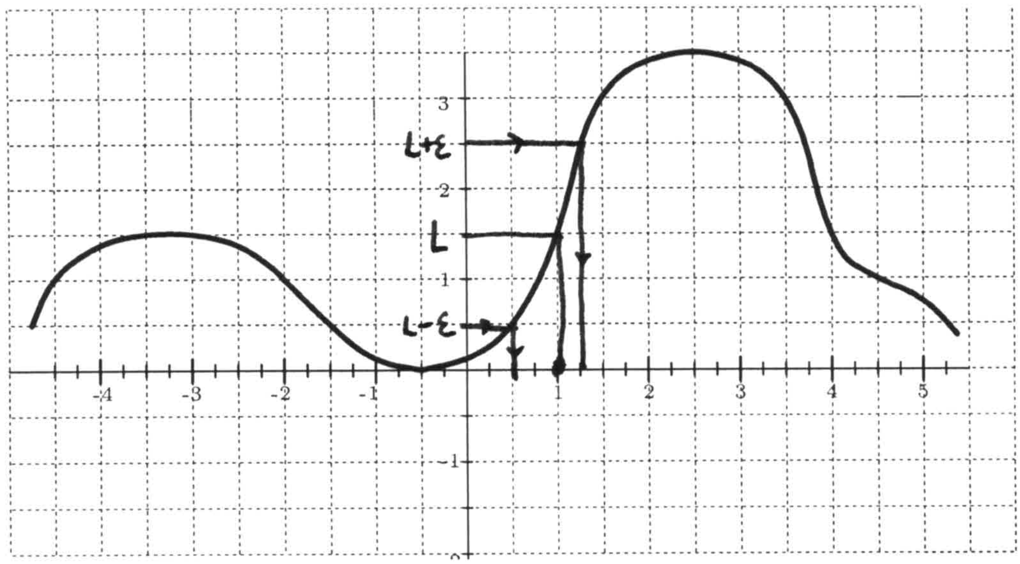
$\approx$ TINY

$= 0.$



(b) ____________________

8:41



5. (a) [4 points] Suppose that $f(x)$ is defined using the graph above.
How close must x be to 1 to ensure that $f(x)$ is within 1 units of 1.5? You must support your answer **by what you draw** in the figure.

If x is within $\frac{1}{4} = 0.25$ of 1,
Then $f(x)$ is within 1 unit of 1.5

(b) [4 points] Let $f(x) = 4x$
Find a value of δ such that as long as $0 < |x - 2| < \delta$, then $|f(x) - 8| < 0.1$
You must **show your work.**

want:

$$|f(x) - 8| < 0.1$$

$$|4x - 8| < 0.1$$

$$|4(x-2)| < 0.1$$

$$|4| \cdot |x-2| < 0.1 = \frac{1}{10}$$

$$|x-2| < \frac{1}{40}$$

let $\delta = \frac{1}{40}$

If x is within $\frac{1}{40}$ of 2,

Then $f(x)$ is within $0.1 = \frac{1}{10}$ of 8

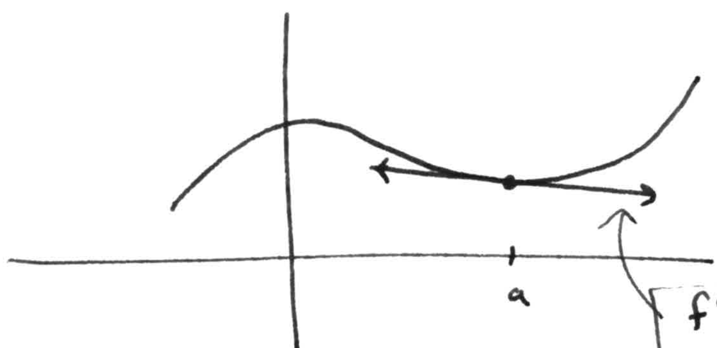
8:44

6. The definition and meaning of the derivative.

(a) [4 points] Write down the limit definition of the derivative of the function $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) [4 points] Explain the graphical meaning of $f'(a)$ with words and with a sketch.



$f'(a)$ is the slope of $f(x)$ at a.

$f'(a)$ is the **slope** of this line

(c) [4 points] Let $f(x) = x^2 - x + 2$. Find $f'(1)$ using the *limit definition* of the derivative.

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$f(1+h) = (1+h)^2 - (1+h) + 2$$
$$= \cancel{1} + 2h + h^2 - \cancel{1} - h + 2$$
$$= h + h^2 + 2$$
$$f(1) = 1^2 - 1 + 2 = 2$$

$$= \lim_{h \to 0} \frac{(h^2 + h + \cancel{2}) - (\cancel{2})}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(h+1)}{\cancel{h}}$$

$$= \lim_{h \to 0} (h+1) = 1$$

8:42

7. From this question onward, you may use the derivative rules. Show **major** steps for credit.

(a) [5 points] Let $f(x) = 3x + 4\sin(x) - x^2 + 2$. Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[3x + 4\cdot\sin(x) - x^2 + 2\right]$$

$$= 3 + 4\cdot\cos(x) - 2x$$

(a) ______________________

(b) [5 points] Compute the derivative of

$$f(x) = \frac{2x}{\cos(x)}$$

$$f'(x) = \frac{d}{dx}\left[\frac{2x}{\cos(x)}\right] = \frac{\cos(x)\cdot\frac{d}{dx}[2x] - 2x\cdot\frac{d}{dx}[\cos(x)]}{(\cos(x))^2}$$

$$= \frac{2\cdot\cos(x) - 2x\cdot(-1)\cdot\sin(x)}{(\cos(x))^2}$$

(b) $f'(x) = \dfrac{2\cdot\cos(x) + 2\cdot x\cdot\sin(x)}{(\cos(x))^2}$

(c) [5 points] Compute $\frac{d}{dx}\left[\sqrt{1-x^2}\right]$

$$= \frac{d}{dx}\left[(1-x^2)^{\frac{1}{2}}\right]$$

$$= \frac{1}{2}\cdot(1-x^2)^{-\frac{1}{2}}\cdot\frac{d}{dx}\left[1-x^2\right]$$

$$= \frac{1}{\cancel{2}}\cdot(1-x^2)^{-1/2}\cdot(-\cancel{2}x)$$

(c) = ______________________

$$= \frac{-x}{\sqrt{1-x^2}}$$

8.49

8. Let $f(x)$ be the function

$$f(x) = \frac{1}{2x-1}$$

(a) [5 points] Find the *slope* of the tangent to $f(x)$ at the point with $a = 1$.

Slope of tangent to f(x) at 1 $= f'(1)$

$$f'(x) = \frac{d}{dx}\left[(2x-1)^{-1}\right] = (-1)(2x-1)^{-2} \cdot \frac{d}{dx}[2x-1]$$

$$= (-1)\frac{1}{(2x-1)^2} \cdot 2$$

$$f'(x) = \frac{-2}{(2x-1)^2}$$

$$f'(1) = \frac{-2}{(2\cdot 1-1)^2} = \frac{-2}{(1)^2} = -2$$

(a) ______________________

(b) [5 points] Find the *equation* of tangent line to $f(x)$ at the point with $a = 1$.

eqn of line: $y = m(x - x_1) + y_1$

for tangent line at a

$x_1 = 1$

$y_1 = f(1) = \frac{1}{2\cdot 1 - 1} = \frac{1}{1} = 1$

& $m = f'(1) = -2$

tangent line is: $y = (-2)(x-1) + 1$

(b) ______________________

9. Let $f(x)$ be the function

$$f(x) = e^x(x - 3) = x \cdot e^x - 3e^x$$

(a) [5 points] Find a formula for $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[x \cdot e^x\right] - 3\frac{d}{dx}\left[e^x\right]$$

$$= x \cdot \frac{d}{dx}\left[e^x\right] + e^x \cdot \frac{d}{dx}\left[x\right] - 3e^x$$

$$= x \cdot e^x + e^x - 3e^x$$

$$= x \cdot e^x - 2e^x$$

(a) $f'(x) = x \cdot e^x - 2e^x$

(b) [4 points] Find the tangent line to f at the point $(0, -3)$.

$$y = m(x - x_1) + y_1$$

<u>for tangent</u>

$x_1 = 0$

$y_1 = -3$

$m = f'(0) = 0 \cdot e^0 - 2e^0 = -2$

(b) $y = (-2) \cdot (x - 0) + (-3)$

$y = -2x - 3$

(c) [5 points] Find a formula for $f''(x)$.

$$f''(x) = \frac{d}{dx}\left[x \cdot e^x - 2e^x\right]$$

$$= \frac{d}{dx}\left[x \cdot e^x\right] - 2 \cdot \frac{d}{dx}\left[e^x\right]$$

$$= x \cdot e^x + e^x \cdot 1 - 2e^x$$

$$\boxed{f''(x) = x \cdot e^x - e^x}$$

(c)