Name: ______________________ Section: ______________________

Principles of Sketching[1]

There are four basic elements of a sketch

1. **The Drawing:** Sketch the physical objects being described. Try to match the scale and relations between things.
2. **Annotations:** Add names, labels, and explanatory notes.
   - Label quantities that can change with *letters*. If you are told that a quantity (length, angle, etc) is *fixed*, you can label the drawing with its value.
   - You might also want to add additional lines to create a shape like a triangle, which can be used along with trigonometry or the Pythagorean theorem.
3. **Arrows:** Draw arrows to indicate motion. Once drawn, these arrows can often help you find out where to fill in the missing lines to create a triangle.
4. **Notes:** Next to your drawing, write down any formulas that may be useful for relating the relevant quantities. Common examples are area, volume, trig, and distance formulas. You may also use facts about similar triangles.

The Form of an Optimization Problem

Minimize/maximize	subject to the	**constraint**(s)
A function		some equation(s)

Useful Formulas

1. If a solid has a constant cross-section, its volume equals its surface area times its length.

   For example, the volume of a rectangular prism is $V = bhl$.

2. To find the cost of an object, use your sketch to add up the cost of each side.

   Notice that different sides may have different per-unit costs.

3. The distance between (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

   If you are minimizing distance, it is easier to minimize "distance squared"

$$F = d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

[1] Adapted from §3.4 of *Sketching User Experiences: The Workbook*, by Greenberg et.al.

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For each problem

(a) Sketch a picture of the **given** situation. *Draw* the physical objects described. Then *annotate* the drawing with labels, lines, etc. Make a *note* of any additional information.

(b) What is the function F that you **want** to **optimize**? Do you want a max or min?

What are the **constraints**, expressed in mathematical language?

(c) Use the constraints to rewrite F in terms of a single variable.

(d) Find the desired absolute max/min.

(e) **Verify** that your answer is a max/min.

1. Find two positive numbers whose sum is 25 and whose product is as *small* as possible.

2. Find two numbers x and y such that $xy^2 = 54$ and which minimizes $F = x^2 + y^2$.

3. You are building a rectangular garden. You are tired of your neighbors stealing your produce and you have looked into several types of electric fence. The material for the side of the fence near your house is \$1 per foot. The other three sides will cost \$2 per foot. If you have \$100, what is the maximum amount of area you can protect?

4. You want to turn part of your yard into a dog park. For zoning reasons, you want the park to be exactly 100 ft^2. Suppose that your house will form the west boundary of the yard, that the fence for the north boundary will cost \$3 per foot and that the other two sides will cost \$1 per foot. What are the dimensions of the yard with the cheapest surrounding fence?

5. You are designing a rectangular display box with an open top. The box should have a volume of 2 m^3, and the length of the base should equal twice the width of the base. The material for the base costs \$5 per square meter, and the material for the sides costs \$10 per square meter. Find the cost of the materials for the cheapest container.

6. You are designing a rectangular display box with an open top. The width of the base must equal twice its length. The material for the base costs \$6/in^2 and the material for the sides costs \$2/in^2. What is the largest volume box that you can make for \$120?

7. You are crossing a 2 mile wide river. Your camp is on the other side, 5 miles downstream. You are hungry, and you want to get there as quickly as possible. If you can swim at 3 mi/hr and hike at 5 mi/hr, where should you land to get there the fastest?