Instructions:

- This exam contains 10 pages. When we begin, check you have *one* of each page.
- You will have 70 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.** In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:
By writing my name below, I agree that:
All the work which appears on this exam is entirely my own.
I will not look at other peoples' work,
and I will not communicate with anyone else during the exam.
I will not use any calculators, notes, etc.
I understand that violating the above carries *serious consequences*,
both moral and academic.

Printed Name: Key Signature: ____________

Section: ____________

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	8	8	10	15	8	12	15	9	15	100
Score:										

1. Answer the following quesitons about functions.

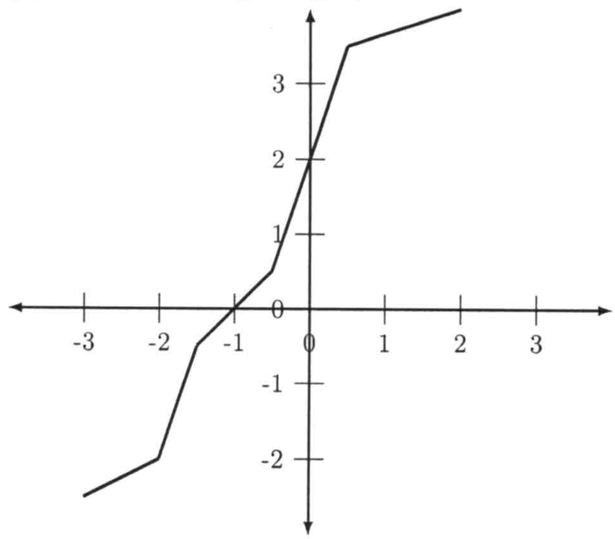
   (a) [4 points] Let $f(x) = x^2 + 2x + 1$ and $g(x) = \sqrt{x}$.
   Write down $(f \circ g)(x)$, and simplify where it is reasonable.

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 + 2(\sqrt{x}) + 1$$

$$= x + 2\sqrt{x} + 1$$

(a) ______________________

   (b) [4 points] Suppose $f(x)$ is defined using the graph below.



Explain why

i. [redacted] f is a function.

it passes the vertical line test

Explain why

ii. [redacted] f has an inverse.

it passes the horizontal line test

iii. Find $f^{-1}(2)$.

$f^{-1}(2) = x$

$\Leftrightarrow$

$2 = f(x)$

$\Leftrightarrow$

$x = 0$

Because

$f(0) = 2$ ✓

iii. $f^{-1}(2) = 0$

2. Exponential and logarithmic properties

(a) [2 points] Simplify the expression:

$$\frac{x^3 \cdot (7x)^{-1}}{(x^{12})^2} = \frac{x^3}{x^{24} \cdot 7 \cdot x} = \frac{x^3}{x^{25} \cdot 7}$$

$$= \frac{1}{x^{22} \cdot 7}$$

(a) $\frac{1}{7x^{22}}$

(b) [2 points] Express as a single logarithm:

$$\ln(3) + 2\ln(5)$$

$$\begin{array}{r} 25 \\ 3 \\ \hline 75 \end{array}$$

$$= \ln(3) + \ln(5^2)$$

$$= \ln(3 \cdot 5^2) = \ln(75)$$

Both ok

(b)

(c) [2 points] Simplify the expression:

$$\ln(e^{\ln(e^{12})}) = \ln(e^{12}) = 12$$

(c) 12

(d) [2 points] Solve for x:

$$e^{2x+1} = 3$$

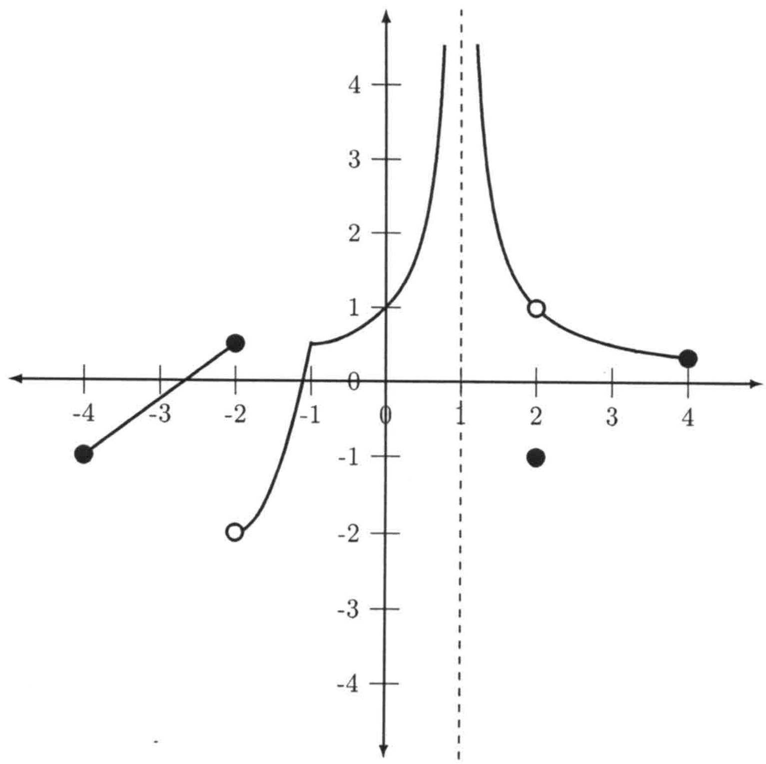
$$\ln(e^{2x+1}) = \ln(3)$$

$$2x+1 = \ln(3)$$

$$2x = \ln(3) - 1$$

(d) $x = \frac{\ln(3)-1}{2}$

3. Suppose that $f(x)$ is defined using the following graph.



(a) [6 points] List the points where f is **not** continuous, and describe the type of discontinuity.

$x = -2$ — jump

$x = 1$ — asymptote

$x = 2$ — removable discontinuity

(b) [4 points] Where is $f(x)$ **not** differentiable? That is, for what x is $f'(x)$ undefined?

$x = -2$

$x = -1$

$x = 1$

$x = 2$

4. Evaluate the following limits. Be sure to **show all work.**

(a) [5 points] $\lim_{x\to 2-} \frac{x^2-4x+3}{x^2-5x+6}$

Can I plug in 2?
bottom is $2^2-5\cdot2+6 = 4-10+6=0$
No - I cannot

$$= \lim_{x\to 2^-} \frac{(x-1)\cancel{(x-3)}}{(x-2)\cancel{(x-3)}}$$

$$= \lim_{x\to 2^-} \frac{x-1}{x-2}$$

still cannot plug in 2

as $x\to 2^-$, $(x-2)\to 0^-$ & $(x-1)\to 1^-$

so as $x\to 2^-$, $\frac{1}{(x-2)}\to -\infty$ and so $\frac{x-1}{x-2}\to -\infty$

(a) the limit is $-\infty$

(b) [5 points] $\lim_{x\to 1-} \frac{x^2-3x+2}{x^2+7x+6}$

Can I plug in 1?
bottom is $1^2+7\cdot1+6 = 1+7+6 \neq 0$
This is a rational function, So is continuous where defined!
$\Rightarrow$ Yes! I can plug in 1

$$= \frac{1^2-3\cdot1+2}{1^2+7\cdot1+6}$$

$$= \frac{1-3+2}{1+7+6} = \frac{0}{14} = 0$$

(b) 0

(c) [5 points] $\lim_{x\to -1} \frac{x^2+4x+3}{x^2+3x+2}$

Can I plug in (-1)?
bottom is $(-1)^2+3(-1)+2 = 1-3+2=0$
$\Rightarrow$ No I cannot

$$= \lim_{x\to -1} \frac{\cancel{(x+1)}(x+3)}{\cancel{(x+1)}(x+2)}$$

$$= \lim_{x\to -1} \frac{(x+3)}{(x+2)}$$

I can plug -1 in here

$$= \frac{(-1)+3}{(-1)+2} = \frac{2}{1} = 2$$

(c) 2

5. Evaluate the following limits. Be sure to **show all work**.

(a) [4 points] $\lim_{x\to\infty} \frac{x^2 - 3x + 2}{x^2 + 7x + 6}$ $= \lim_{x\to\infty} \frac{\cancel{x^2}(1 - \frac{3}{x} + \frac{2}{x^2})}{\cancel{x^2}(1 + \frac{7}{x} + \frac{6}{x^2})}$

$$= \lim_{x\to\infty} \frac{1 - \overset{\to 0}{\frac{3}{x}} + \overset{\to 0}{\frac{2}{x^2}}}{1 + \underset{\downarrow 0}{\frac{7}{x}} + \underset{\downarrow 0}{\frac{6}{x^2}}}$$

$$= 1$$

(a) 1

(b) [4 points] $\lim_{x\to\infty} \frac{\sqrt{3x^8 + 2}}{x^3 + 6}$ $= \lim_{x\to\infty} \frac{\sqrt{x^8(3 + \frac{2}{x^8})}}{x^3(1 + \frac{6}{x^3})}$

$$\frac{x}{1} = \lim_{x\to\infty} \frac{x^4\sqrt{3 + \frac{2}{x^8}}}{x^3(1 + \frac{6}{x^3})}$$

$$= \lim_{x\to\infty} \underset{\infty}{\boxed{x}} \; \underset{3}{\boxed{\frac{\sqrt{3 + \overset{\to 0}{\frac{2}{x^8}}}}{(1 + \underset{\to 0}{\frac{6}{x^3}})}}}$$

(b) ∞

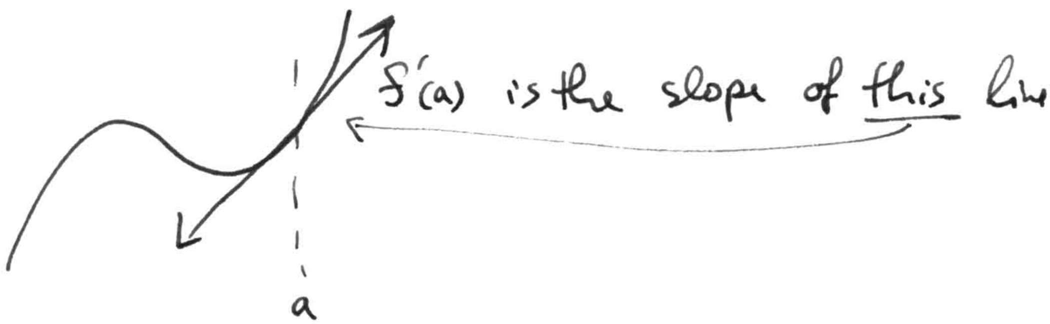
6. The definition and meaning of the derivative.

(a) [4 points] Write down the limit definition of the derivative of the function $f(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) [4 points] Explain the graphical meaning of $f'(a)$ with words and with a sketch.

$f'(a)$ is the slope of the tangent to $f(x)$ at the point $(a, f(a))$



(c) [4 points] Let $f(x) = x^2 + x + 1$. Find $f'(1)$ using the *limit definition* of the derivative.

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{[(1+h)^2 + (1+h) + 1] - [1^2 + 1 + 1]}{h}$$

$$= \lim_{h \to 0} \frac{[1 + 2h + h^2 + 1 + h + 1] - 3}{h} = \lim_{h \to 0} \frac{3h + h^2}{h}$$

$$= \lim_{h \to 0} (3 + h) = 3$$

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7. Evaluate the derivatives below. You may use the derivative rules, but **show your work.**

(a) [5 points] Compute the derivative of

$$f(x) = e^x \cdot (\sqrt{x} + x) \qquad (fg)' = fg' + gf'$$

$$f'(x) = \frac{d}{dx}\left(e^x(\sqrt{x}+x)\right) = e^x \cdot \frac{d}{dx}(\sqrt{x}+x) + (\sqrt{x}+x)\cdot\frac{d}{dx}(e^x)$$

$$= e^x\left(\tfrac{1}{2}x^{-\frac{1}{2}}+1\right) + (\sqrt{x}+x)\cdot e^x$$

(a) $\boxed{e^x\left(\tfrac{1}{2}\cdot x^{-\frac{1}{2}} + 1 + \sqrt{x} + x\right)}$

(b) [5 points] Compute the derivative of

$$f(x) = \sin(x) \cdot \sin(x) \qquad (fg)' = fg' + gf'$$

$$f'(x) = \frac{d}{dx}\left(\sin(x)\cdot\sin(x)\right) = \sin(x)\cdot\frac{d}{dx}(\sin(x)) + \sin(x)\cdot\frac{d}{dx}(\sin(x))$$

$$= \sin(x)\cdot\cos(x) + \sin(x)\cdot\cos(x)$$

(b) $\boxed{2\cdot\sin(x)\cdot\cos(x)}$

(c) [5 points] Compute the derivative of

$$f(x) = \frac{\cos(x) - 1}{x} \qquad \boxed{\left(\frac{t}{b}\right)' = \frac{b\cdot t' - t\cdot b'}{b^2}}$$

$$f'(x) = \frac{d}{dx}\left(\frac{\cos(x)-1}{x}\right) = \frac{x\cdot\frac{d}{dx}(\cos(x)-1) - (\cos(x)-1)\cdot\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x(-\sin(x)) - (\cos(x)-1)}{x^2} = \boxed{\frac{-x\sin(x) - \cos(x) - 1}{x^2}}$$

(c) __________

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8. Let $f(x)$ be the function

$$f(x) = (x+1)(5-2x) = 5x - 2x^2 + 5 - 2x = \boxed{-2x^2+3x+5}$$

(a) [5 points] Find the *slope* of the tangent to $f(x)$ at the point with $x = 2$.

$$f'(x) = \frac{d}{dx}\left((x+1)\cdot(5-2x)\right) = \frac{d}{dx}\left(-2x^2+3x+5\right)$$

$$= -2\cdot\frac{d}{dx}(x^2) + 3\frac{d}{dx}(x) + \frac{d}{dx}(5)$$

$$= -2\cdot 2x + 3 + 0$$

$$= -4x + 3$$

$$f'(2) = -4(2) + 3 = -8 + 3 = -5$$

(a) -5

(b) [4 points] Find the *equation* of tangent line to $f(x)$ at the point $(2, 3)$.

$$y = m(x - x_1) + y_1$$

$$y = m(x-2) + 3$$

$$y = -5(x-2) + 3$$

or

$$y = -5x + 10 + 3 = -5x + 13$$

when $x = 2$

$$f(2) = (2+1)(5-2\cdot 2)$$

$$= (2+1)(5-4)$$

$$= 3$$

(b) $y = -5(x-2) + 3$

9. Let $f(x)$ be the function

$$f(x) = 8\sqrt{x} - x$$

(a) [5 points] Compute $f'(x)$.

$$f'(x) = \frac{d}{dx}\left(8\sqrt{x} - x\right) = \frac{d}{dx}(8\sqrt{x}) - \frac{d}{dx}(x)$$

$$= 8 \cdot \frac{1}{2} x^{-\frac{1}{2}} - 1$$

$$\boxed{f'(x) = 4 \cdot x^{-\frac{1}{2}} - 1 \quad \text{or} \quad f'(x) = \frac{4}{\sqrt{x}} - 1}$$

(a) $f'(x) = \frac{4}{\sqrt{x}} - 1$

(b) [5 points] Find the tangent line to f at the point [redacted] (1,7)

$x = 1 \Rightarrow f(1) = 8\sqrt{1} - 1 = 8 - 1 = 7$

$x = 1 \Rightarrow f'(1) = \frac{4}{\sqrt{1}} - 1 = 4 - 1 = 3$

$y = m(x-1) + 7$

$y = 3(x-1) + 7$

(b) $y = 3(x-1) + 7$

(c) [5 points] Find the tangent line to f at the point with $(4, 12)$.

$$f'(4) = \frac{4}{\sqrt{4}} - 1 = \frac{4}{2} - 1 = 2 - 1 = 1$$

$y = m(x-4) + 12$

$y = 1(x-4) + 12$

(c)

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