

## Worksheet Problem #1

An airplane flies over a radar station...

(a) Given: ① the plane is flying horizontally (altitude = 3 mi) over the station

~~altitude = 3 mi~~

~~known~~

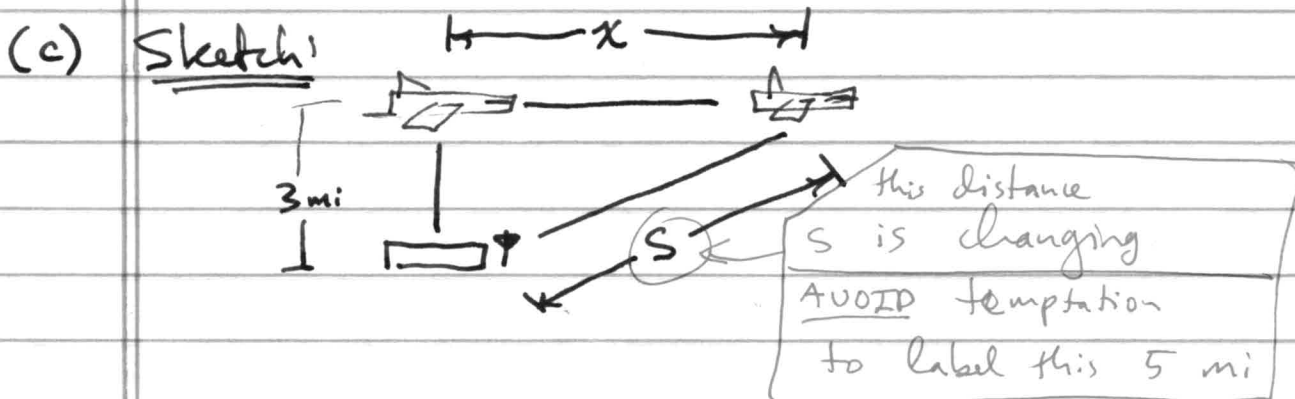
NOTATION: let  $s$  denote the distance between plane and station at time  $t$

② know: when  $s = 5$  mi,  $\frac{ds}{dt} = 500$  mi/hr

(b) Unknown: Ground speed of the airplane.

NOTATION: let  $x$  denote the horizontal distance travelled. at time  $t$

then  $\frac{dx}{dt} =$  ground speed



(d) Equation:

$$3^2 + x^2 = s^2$$

(e) finish the problem:

to relate the rates, we take  $\frac{d}{dt}$  of both sides

$$\frac{d}{dt} (3^2 + x^2) = \frac{d}{dt} (s^2)$$

$$0 + 2 \cdot x \cdot \frac{dx}{dt} = 2 \cdot s \cdot \frac{ds}{dt}$$

so

$$x \cdot \frac{dx}{dt} = s \cdot \frac{ds}{dt}$$

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to find  $\frac{dx}{dt}$ , need:  $s, x$ , &  $\frac{ds}{dt}$ . HAVE:  $s$  &  $\frac{ds}{dt}$  already

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when  $s = 5$  mi, we can compute  $x$

$$3^2 + x^2 = 5^2$$

$$9 + x^2 = 25$$

$$x^2 = 16$$

$$x = 4$$

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so when  $s = 5$  mi,

$$x \cdot \frac{dx}{dt} = s \cdot \frac{ds}{dt}$$

$$4 \cdot \frac{dx}{dt} = 5 \cdot (500)$$

$$\frac{dx}{dt} = \frac{5 \cdot 500}{4} = 625 \text{ mi/hr}$$

## Worksheet Problem #2

An ice cube melts, ...

(a) GIVEN: the surface area's rate of change

NOTATION: write  $a$  for the cube's surface area  
at time  $t$

we are given:



$$\frac{da}{dt} = -3 \frac{\text{in}^2}{\text{s}}$$

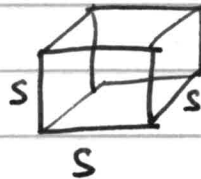


the surface area is getting smaller.

(b) Unknown: the side length's rate of change

notation:  $s$  is the side length at time  $t$ .

(c) Sketch



(d) note: the cube has 6 equal sides  
each of area  $s^2$

$$\Rightarrow a = 6 \cdot s^2$$

(e) complete the problem

relate  $\frac{da}{dt}$  and  $\frac{ds}{dt}$  by

$$\frac{d}{dt}(a) = \frac{d}{dt}(6 \cdot s^2)$$

$$\frac{da}{dt} = 2 \cdot 6 \cdot s \cdot \frac{ds}{dt}$$

$$\frac{da}{dt} = 12 \cdot s \cdot \frac{ds}{dt}$$

$$\text{when } s = 1'', \quad \frac{da}{dt} = -3 \frac{\text{in}^3}{\text{s}}$$

$$-3 = 12 \cdot 1 \cdot \frac{ds}{dt}$$

$$\text{so } \boxed{\frac{ds}{dt} = \frac{-1}{4} \frac{\text{in}}{\text{sec}}}$$

### Worksheet Problem #3

A streetlight is mounted at the top of a pole <sup>6 meters</sup> ...

(a) Given: height of light = 6 m

a person is walking toward it

Notation:  $x$  is the person's distance to the light pole <sup>bottom of</sup> at time  $t$

then

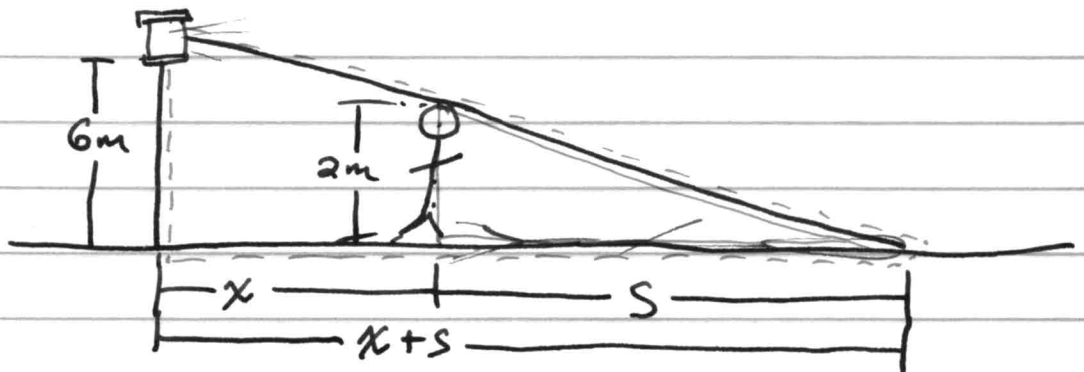
$$\frac{dx}{dt} = -2 \frac{m}{s}$$

because  $x$  is shrinking.

(b) Unknown: how fast their shadow's length is changing

NOTATION:  $s$  is the shadow's length at time  $t$

(c) sketch:



(d) equation we have 2 similar triangles

$$\text{So } \frac{s}{2} = \frac{x+s}{6}$$

(e) finish the problem:  $\frac{s}{2} = \frac{s+x}{6}$

So:  $2s + 2x = 6s$

~~scribble~~

So  $2x = 4s$

$x = 2s$

To relate the rates

$$\frac{d}{dt}(x) = \frac{d}{dt}(2s)$$

$$\frac{dx}{dt} = 2 \cdot \frac{ds}{dt}$$

Because  $\frac{dx}{dt}$  is constant,

~~scribble~~ no matter how far they are from the light

$$\frac{ds}{dt} = \frac{1}{2} \frac{dx}{dt} = \frac{1}{2} (-2) = -1 \frac{m}{s}$$

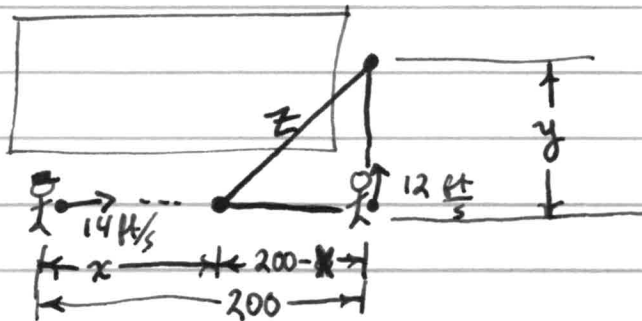
## Worksheet Problem #4

A police officer is walking down a street...

- (a) Given: starting distance = 200 ft  
police officer runs at  $12 \frac{\text{ft}}{\text{s}}$   
felon runs at  $9 \frac{\text{ft}}{\text{s}}$

- (b) Unknown: how fast the straight-line distance is changing

(c)



notation:  $x$  = distance travelled by officer } know  
at time  $t$  }  $\frac{dx}{dt}$

$y$  = distance travelled by felon } know  
at time  $t$  }  $\frac{dy}{dt}$

$z$  = straight-line distance } WANT  
at time  $t$  }  $\frac{dz}{dt}$

(e) equation

$$(200-x)^2 + y^2 = z^2$$

(f) to relate the rates

$$\frac{d}{dt} \left( (200-x)^2 + (y)^2 \right) = \frac{d}{dt} (z^2)$$

$$\frac{d}{dt} \left( (200-x)^2 \right) + 2 \cdot y \cdot \frac{dy}{dt} = 2 \cdot z \cdot \frac{dz}{dt}$$

$$2 \cdot (200-x) \cdot \frac{d}{dt}(200-x) + 2y \cdot \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$2(200-x) \left( -\frac{dx}{dt} \right) + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$-(200-x) \frac{dx}{dt} + y \frac{dy}{dt} = z \frac{dz}{dt}$$

~~Notice because velocity is constant~~

~~$x = 12t$~~

Notice because velocity is constant

$$x = 12t$$

and

$$y = 9$$

when  $t = 10$  s

$$x = 120$$

$$\text{and } y = 90$$

$$\textcircled{\text{so}} \quad z^2 = (120)^2 + (9)^2$$

$$z = 150$$

$$\textcircled{\text{so}} \quad -(200-120) \cdot \left( \frac{12}{s} \right) + (90) \cdot \left( \frac{9}{s} \right) = 150 \cdot \frac{dz}{dt}$$

$$\text{Solving for } \frac{dz}{dt} = \dots = \frac{-287}{150} \approx -5.74 \frac{\text{ft}}{\text{s}}$$



## Worksheet Problem #5:

Suppose there is a trough of water ...

- (a) Given: → dimensions & shape of trough  
→ rate at which water is added

NOTATION:  $V =$  volume at time  $t$

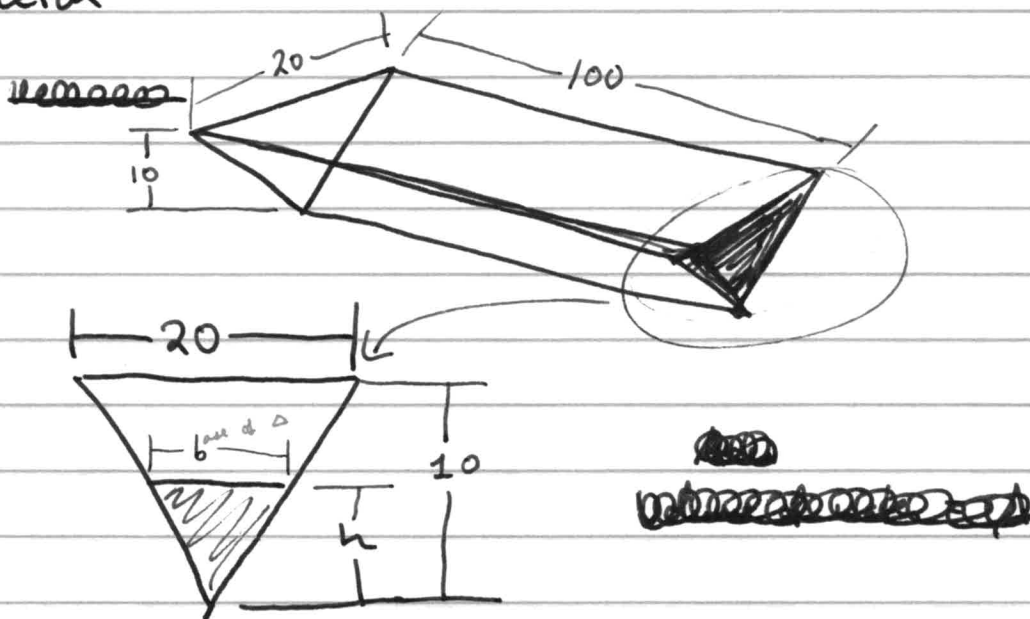
Given:  $\frac{dV}{dt} = 400 \frac{\text{cm}^3}{\text{s}}$

- (b) Unknown: ~~rate of change of water height~~ rate of change of water height  
~~rate of change of water height~~ water at time  $t$ .

NOTATION:  $h =$  height at time  $t$

want:  $\left(\frac{dh}{dt}\right)$ .

- (c) sketch



Note: ① (Volume of water) = (end area) · (length)

② (end area) =  $\frac{1}{2} b \cdot h$

and ③  $\frac{b}{h} = \frac{20}{10}$

← we know nothing about  $b$   
So we must get rid of it

(d) equation:

$$\frac{b}{h} = \frac{20}{10} \Rightarrow b = 2h$$

$$\text{so end area} = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot (2h) \cdot h = h^2$$

$$\text{so volume} = (\text{end area}) \cdot (\text{length}) = 100 \cdot h^2$$

to relate the rates

$$\frac{d}{dt}(V) = \frac{d}{dt}(100 \cdot h^2)$$

$$\frac{dV}{dt} = 100 \cdot h \cdot \frac{dh}{dt}$$

~~when~~

the tank is half full when  $h = \frac{10}{2} = 5$ .

When  $h = 5$

$$\frac{dV}{dt} = 100 \cdot h \cdot \frac{dh}{dt}$$

$$400 = 100 \cdot 5 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{5} \frac{\text{cm}}{\text{s}}$$

## Worksheet Problem #6

Suppose the water trough leaks.

Then

$$\frac{dV}{dt} = \left( \begin{array}{c} \text{rate of} \\ \text{water in} \end{array} \right) - \left( \begin{array}{c} \text{rate of} \\ \text{water out} \end{array} \right)$$

$$= 400 - 100$$

$$\frac{dV}{dt} = 300 \frac{\text{cm}^3}{\text{s}}$$

The rest of the problem is the same  
as it was in #5