Instructions:

- This exam contains 11 pages. When we begin, check you have *one* of each page.
- You will have 75 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.** In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:

By writing my name below, I agree that all the work which appears on this exam is entirely my own.

I will not look at other peoples' work, and I will not communicate with anyone else about the exam.

I will not use any calculators, notes, etc.

I understand that violating the above carries *serious consequences*, both moral and academic.

Printed Name: Key Signature: ______________

Section: ______________

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	12	12	12	6	12	12	10	6	6	12	100
Score:											

1. [12 points] Elasticity

(a) Suppose that the price-demand function for giant paperclips is given by $x(p) = 900 - 3p^2$. Find the elasticity of demand function $E(p)$.

1pt

$$E(p) = \frac{-p}{x} \cdot \frac{dx}{dp}$$

$$x = 900 - 3p^3$$

$$\Rightarrow \frac{dx}{dp} = -3 \cdot 2p$$

$$= \frac{-p}{900-3p^2}(-6p)$$

2pt

$$E(p) = \frac{6p^2}{900-3p^2}$$

(b) Find the price where giant paperclips [redacted] are unit elastic.

$E(p) = 1$ when

2pt

$$1 = \frac{6p^2}{900-3p^2}$$

$$900-3p^2 = 6p^2$$

$$900 = 9p^2$$

$$p^2 = 100$$

3pt

$$p = \pm\sqrt{100}$$

only $\sqrt{100}$ makes sense

$= \cancel{\$}10$

(c) If the current price is \$1, what is the current elasticity? How will a 20% increase in price impact demand?

2pt

$$E(1) = \frac{6 \cdot 1^2}{900 - 3 \cdot 1^2} = 0.006689$$

2pt

20% increase in price $\approx (0.006689)(20\%) = 0.13378\%$ decrease in demand.

2. [12 points] Let $f(x) = \frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}$

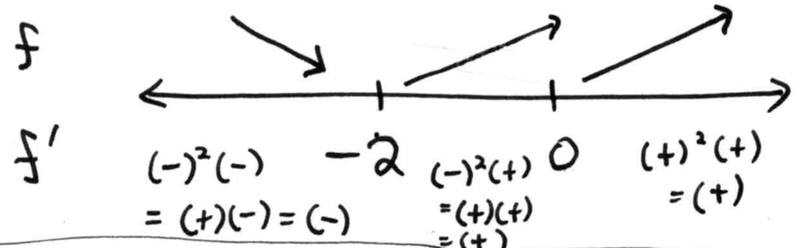
(5 total) (a) Find the intervals where f is increasing and decreasing

$$f'(x) = x^3 + 2x^2 = x^2(x+2)$$

$f'(x) = 0$ when $x = 0$ or -2

$f'(x)$ DNE never

2pt



decreasing on $(-\infty, -2)$

increasing on $(-2, 0) \cup (0, \infty)$

1pt each

(b) Find the intervals where f is concave up and concave down

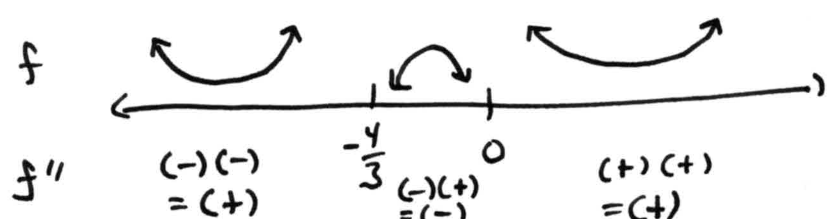
$$f''(x) = 3x^2 + 4x = x(3x+4)$$

4

$f''(x) = 0$ when $x = 0$ or $-\frac{4}{3}$

$f''(x)$ DNE never

1pt



Concave up on $(-\infty, -\frac{4}{3}) \cup (0, \infty)$

Concave down on $(-\frac{4}{3}, 0)$

1pt each

(c) Find the numbers x where the are local maxima and minima. Write DNE if no such point exists.

2

local min at $x = -2$

No local max

(d) Find the numbers x where the is a point of inflection. Write DNE if no such point exists.

1

Inflection pt at $x = \frac{-4}{3}$ and at $x = 0$

3. Compute the following limits. You must **show your work** for full credit.

(a) [6 points] Compute

$$\lim_{x\to\infty} \frac{3+5x^2-2x}{9x^2+4x-2}$$

x is getting bigger & biger positive #

3pt $$= \lim_{x\to\infty} \frac{\cancel{x^2}\left(\frac{3}{x^2}+5-\frac{2}{x}\right)}{\cancel{x^2}\left(9+\frac{4}{x}-\frac{2}{x^2}\right)}$$

$\frac{3}{x^2} \to 0$, $\frac{2}{x} \to 0$, $\frac{4}{x} \to 0$, $\frac{2}{x^2} \to 0$

$$\boxed{\text{think } \frac{3}{(\text{billion})^2} \approx \text{TINY}}$$

3pt $$= \frac{5}{9}$$

(b) [6 points] Compute

$$\lim_{x\to\infty} \frac{3x^2-2}{4x+1}$$

2pt $$= \lim_{x\to\infty} \frac{x^2\cdot\left(3-\frac{2}{x^2}\right)}{x\left(4+\frac{1}{x}\right)}$$

$\frac{2}{x^2} \to 0$, $\frac{1}{x} \to 0$

2pt $$= \lim_{x\to\infty} x\cdot\left(\frac{3}{4}\right)$$

$$\boxed{\text{Think } \approx (\text{us debt})\cdot\frac{3}{4} \approx \text{HUGE}}$$

2pt $$= \infty$$

4. Compute the following limits. You must **show your work** for full credit.

   (a) [6 points] Compute

$$\lim_{x\to\infty} -9x^3 + 3x^2 + 5$$

2pt

$$= \lim_{x\to\infty} x^3\left(-9 + \frac{3}{x} + \frac{5}{x^3}\right)$$

$\frac{3}{x} \to 0$, $\frac{5}{x^3} \to 0$

2pt

<u>Think</u>

$$\approx (\text{HUGE}_{\text{POS}})^3(-9 + 0 + 0)$$

$$\approx \text{Huge Neg}$$

2pt

$$= -\infty$$

5. [12 points] The inventory management cost of a certain production line is given by

$$f(x) = 10 \cdot \frac{1}{x} + x$$

Find the *absolute maximum* **and** the *absolute minimum* costs *on the interval* [1, 20].

Find exact numbers using calculus, and use your calculator to give a decimal approximation.

$$f'(x) = \frac{d}{dx}\left[10 \cdot x^{-1} + x\right]$$

$$= 10 \cdot (-1)\, x^{-2} + 1$$

3pt $f'(x) = \dfrac{-10}{x^2} + 1$

$f'(x) = 0$ when

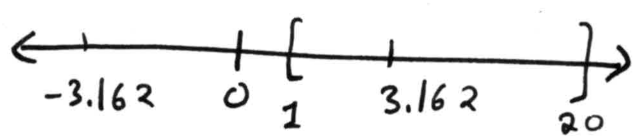
$$\frac{-10}{x^2} + 1 = 0$$

$$1 = \frac{10}{x^2}$$

$$x^2 = 10$$

2pt $x = \pm\sqrt{10} = \pm\ 3.162$

2pt $f'(x)$ DNE when $x = 0$



check endpoints & critical #'s

1pt $f(1) = 10 \cdot \frac{1}{1} + 1 = 11$

2pt $f(3.162) = 10 \cdot \frac{1}{3.162} + 3.162 = 6.325$

2pt $f(20) = 10 \cdot \frac{1}{20} + 20 = 20.5$

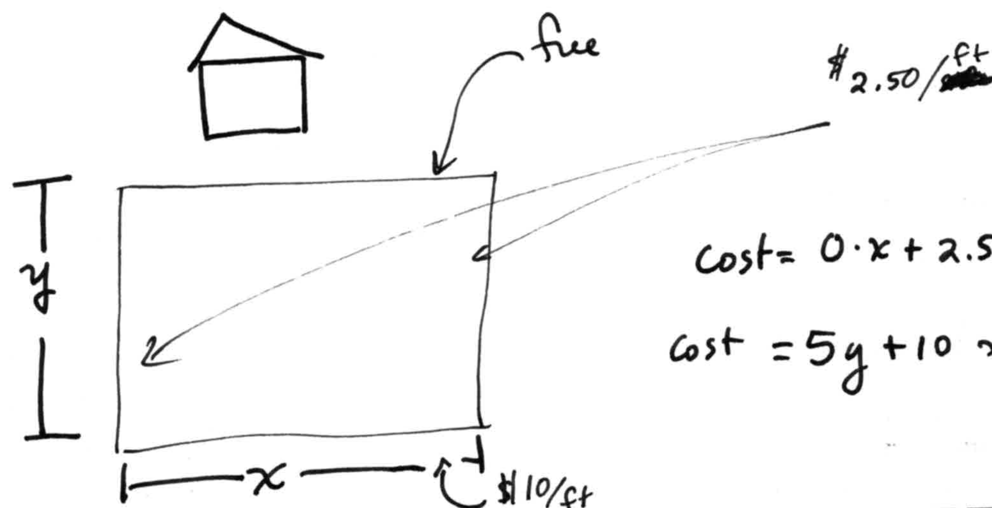
$\boxed{\text{absolute min on } [1,20] \text{ at } \sqrt{10}}$

$\boxed{\text{absolute max on } [1,20] \text{ at } 20}$

6. [12 points] You are planning on building an outdoor play area for your home daycare business. You have \$200 that you can use for the the fencing. Your house will form the north boundary of the yard. The south side, which faces the street, will cost \$10 per foot. The other two sides will cost \$2.50 per foot.

Find the dimensions of the largest area play area that you can build.

You must **show all work**, including verifying that your answer gives the maximum area.



(2pt)

$\text{cost} = 0\cdot x + 2.5y + 10x + 2.5y$

$\text{cost} = 5y + 10x$

maximize area subject to constraint (cost)

$F = xy$

$= x(40 - 2x)$

(2pt) $F(x) = 40x - 2x^2$

maximize this

$200 = 5y + 10x$

(2pt) $5y = 200 - 10x$

$y = 40 - 2x$

(1pt) $F'(x) = 40 - 4x$

F (1pt)

F' (+) 10 (−)

(2pt)

$$\boxed{\begin{array}{l} F'(x) = 0 \text{ when } 40 = 4x \\ \qquad\qquad\qquad\quad x = 10 \\ F' \text{ DNE (never)} \end{array}}$$

(2pt)

$$\boxed{\begin{array}{l} \text{maximum area when} \\ x = 10 \\ y = 40 - 2\cdot 10 = 20 \end{array}}$$

7. [10 points] Suppose that the unit-price function for post-post-modern paintings is given by $p = 405 - x^3$, and that the cost of creating each painting is \$5.

(a) Find revenue, cost, and profit as functions of the number of units sold.

$$R(x) = p \cdot x = (405 - x^3)\, x = 405x - x^4$$

2pt

$$C(x) = 5 \cdot x$$

$$P(x) = R(x) - C(x) = (405x - x^4) - 5x$$

$$= 400x - x^4$$

(b) Use calculus to find the quantity you should sell to make the maximum *profit*. You must show your work.

2pt

$$P'(x) = 400 - 4x^3$$

2pt

$P'(x) = 0$ when

$$400 - 4x^3 = 0$$
$$4x^3 = 400$$
$$x^3 = 100$$
$$x = \sqrt[3]{100}$$

$P'(x)$ DNE (never)

P ↗ ↘

P′ (+) $\sqrt[3]{100}$ (−)

2pt

max profit when

$$x = \sqrt[3]{100} = 4.64$$

(c) What is the maximum profit?

2pt

max profit IS $P(4.64) = 400(4.64) - (4.64)^4$

$= \$1{,}392.48$

8. [6 points] Compute $\int x^3\sqrt{\underbrace{x^4+4}_{u}}\,dx$ $\quad \frac{du}{4x^3}$

$$\boxed{\begin{aligned} u &= x^4+4 \\ \frac{du}{dx} &= 4x^3 \\ \frac{du}{4x^3} &= dx \end{aligned}}$$

2pt

$$= \int \cancel{x^3}\cdot\sqrt{u}\cdot\frac{du}{4\cancel{x^3}}$$

2pt

$$= \int \frac{1}{4}\cdot u^{1/2}\,du$$

$$= \frac{1}{4}\cdot\frac{u^{3/2}}{3/2} + C$$

$$= \frac{1}{4}\cdot\frac{(x^4+4)^{3/2}}{3/2} + C$$

2pt

$$= \frac{(x^4+4)^{3/2}}{6} + C$$

9. [6 points] Compute $\int \frac{x^3+3}{x^2}\,dx$

Too Hard
$\Rightarrow$ must rewrite

$$= \int \frac{x^3}{x^2} + \frac{3}{x^2}\,dx$$

(2pt)

$$= \int x\,dx + \int 3x^{-2}\,dx$$

$$= \frac{x^2}{2} + 3\cdot\frac{x^{-1}}{-1} + C$$

$$= \underbrace{\frac{x^2}{2}}_{2pt} \; \underbrace{-\frac{3}{x}}_{2pt} + C$$

$\boxed{-1 \text{ if missing "+C"}}$

10. (a) [6 points] Compute $\int \frac{1}{x \cdot \left(\ln(x)\right)^2}\, dx$.

2pt

$$\boxed{\begin{aligned} u &= \ln(x) \\ \frac{du}{dx} &= \frac{1}{x} \\ x \cdot du &= dx \end{aligned}}$$

2pt $= \int \frac{1}{\cancel{x} \cdot (u)^2} \cdot \cancel{x}\, du$

$= \int u^{-2}\, du$

2pt $= \frac{u^{-1}}{-1} + C = \boxed{\frac{-1}{\ln(x)} + C}$

(b) [6 points] Suppose that $f'(x) = 12 + 3x^2 + 20x^4$, and that $f(1) = 4$. Find $f(x)$.

$f(x) = 12x + 3 \cdot \frac{x^3}{3} + 20 \cdot \frac{x^5}{5} + C$

2pt $f(x) = 12x + x^3 + 4x^5 + C$

know

$f(1) = 4 = 12 \cdot 1 + 1^3 + 4 \cdot 1^5 + C$

$4 = 12 + 1 + 4 + C$

$C = -13$

2pt

2pt $f(x) = 12x + x^3 + 4x^5 - 13$