Instructions:

- This exam contains 14 pages. When we begin, check you have *one* of each page.
- You will have 2 hours to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.** In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other non-approved electronic devices off now!

Academic Honesty:

By writing my name below, I agree that all the work which appears on this exam is entirely my own.

I will not look at other peoples' work, and I will not communicate with anyone else about the exam.

I will not use any calculators, notes, etc.

I understand that violating the above carries *serious consequences*, both moral and academic.

Printed Name: Key Signature: ____________

Section: ____________

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Points:	10	10	12	12	16	12	12	12	12	12	8	12	10	150
Score:														

1653

1. (a) [5 points] Let $f(x) = \ln(x^2+1)$. Compute $f'(x)$.

$$f'(x) = \frac{1}{x^2+1} \cdot \frac{d}{dx}\left[x^2+1\right]$$

$$= \frac{1}{x^2+1} \cdot 2x$$

$$\boxed{f'(x) = \frac{2x}{x^2+1}}$$

← 5pt

(b) [5 points] Let $y = e^{4x+1}$. Compute $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^{4x+1}\right]$$

$$= e^{4x+1} \cdot \frac{d}{dx}\left[4x+1\right]$$

$$\boxed{\frac{dy}{dx} = 4 \cdot e^{4x+1}}$$

← 5pt

1654

2. [10 points] Suppose you make an investment at 3% interest, compounded monthly.

(a) If your initial investment is \$1,000, give the formula for the balance after t years. Use this to find the balance after 12 years.

$$F(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$F(t) = 1000 \cdot \left(1 + \frac{0.03}{12}\right)^{12t}$$

$$\boxed{F(t) = 1000 \cdot (1.0025)^{12t}}$$ 2pt

$F(12) =$ [redacted]

$$1000\,(1.0025)^{12 \cdot 12}$$

$$\boxed{= \$1432.69}$$ 3pt

(b) How long would it take your investment of \$1,000 to grow to \$12,000?

find t s.t.

$$12{,}000 = 1{,}000 \cdot (1.0025)^{12t}$$

$$12 = (1.0025)^{12t}$$ 2pt

$$\ln(12) = \ln\left((1.0025)^{12t}\right)$$

$$\ln(12) = 12t \cdot \ln(1.0025)$$

$$t = \frac{\ln(12)}{12 \cdot \ln(1.0025)} \approx 82.9 \text{ years}$$ 3pt

1658

3. [12 points] Suppose that the cost function (in \$ per in^3) for incandescent paint is given by

$$C(x) = \sqrt{x+1}$$

(a) Find the marginal cost function.

$$C'(x) = \frac{1}{2} \cdot (x+1)^{-\frac{1}{2}} \cdot \frac{d}{dx}[x+1]$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{x+1}}$$

4pt

(b) Find $C'(8)$ correct to 3 decimal places. What is the business meaning of this?

$$C'(8) = \frac{1}{2} \cdot \frac{1}{\sqrt{8+1}} = \frac{1}{2} \cdot \frac{1}{\sqrt{9}} = \frac{1}{6} = 0.16\overline{6}$$

4pt

(c) Find the cost of the 9^{th} unit correct to 3 decimal places.

$$C(9) - C(8) = \sqrt{9+1} - \sqrt{8+1}$$

$$= \sqrt{10} - \sqrt{9}$$

$$\approx 0.1623$$

4pt

1700

4. [12 points] Suppose that the price-demand function for glossy paper is given by

$$x(p) = 10e^{-2p}$$

(a) Find the elasticity function $E(p)$.

$$E(p) = \frac{-p}{x} \cdot \frac{dx}{dp}$$

$$\frac{dx}{dp} = \frac{d}{dp}\left[10e^{-2p}\right] = 10e^{-2p} \cdot (-2)$$

$$= \frac{-p}{\cancel{10 \cdot e^{-2p}}} \cdot \cancel{10e^{-2p}} \cdot (-2)$$

6pt

$$E(p) = 2p$$

(b) Find the interval where the price is inelastic.

Inelastic when $E(p) < 1$ 1pt

when $2p < 1$

$p < \frac{1}{2}$

Inelastic on $(0, \frac{1}{2})$ 2pt

(c) If the current price is \$0.25, how much will a 10% increase in price impact demand?

$$(\text{\% decrease in demand}) = E(0.25) \cdot (\text{\% increase in price})$$

$$= 2 \cdot (0.25) \cdot 10$$

$$= 0.5 \cdot 10$$

3pt

$= 5$ % decrease in demand

1702

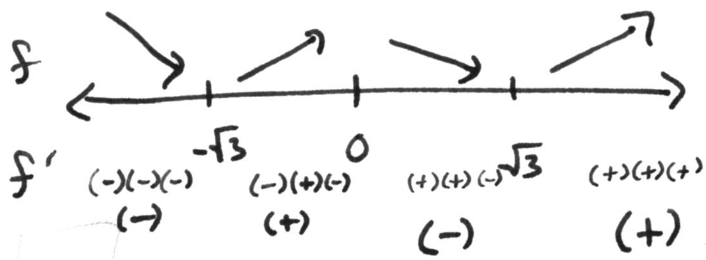
5. [16 points] Let $f(x) = x^4 - 6x^2 + 1$

Find the following (or answer DNE). You must show all work.

1 pt for each answer AND 1 pt for each f' & f'' AND 1 pt for each sign chart

1. Find the intervals where $f(x)$ is increasing/decreasing. Identify which is which.

$$f'(x) = 4x^3 - 12x = 4x(x^2-3) = 4x(x+\sqrt{3})(x-\sqrt{3})$$



/6

Increasing on

$(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

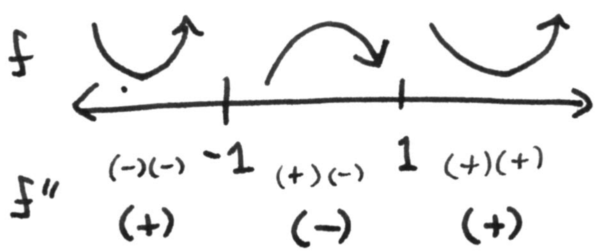
Decreasing on

$(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

2. Find the intervals where $f(x)$ is concave up/down. Identify which is which.

$$f''(x) = 12x^2 - 12 = 12(x^2-1) = 12(x+1)(x-1)$$

/5



concave up on

$(-\infty, -1) \cup (1, \infty)$

concave down on

$(-1, 1)$

3. Find the x value(s) of the local maxima and local minima of f. Identify which is which.

/3

local max at 0

local min at $\sqrt{3}$ & $-\sqrt{3}$

4. Find the x value(s) of the inflection points of f.

inflection at -1 and 1.

/2

6. [12 points] Find the absolute maximum achieved by the function

$$f(x) = 1500\sqrt{x} - 100x \quad \text{for} \quad x \geq 0$$

You must **show your work** and verify this is an absolute maximum.

$$f'(x) = 1500 \cdot \tfrac{1}{2} \cdot x^{-\frac{1}{2}} - 100$$

$$= 750 \cdot \frac{1}{\sqrt{x}} - 100 \quad \leftarrow 5pt$$

Critical #'s

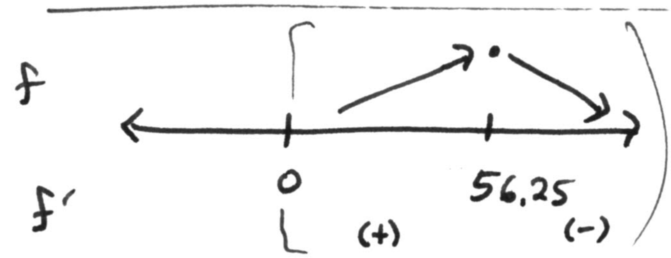
$f'(x)$ DNE when $x = 0$

$f'(x) = 0$ when $\frac{750}{\sqrt{x}} = 100$

when $\sqrt{x} = 7.5$

when $x = 56.25$

5pt



NOT a closed interval
⇒ need to check sign chart

2pt

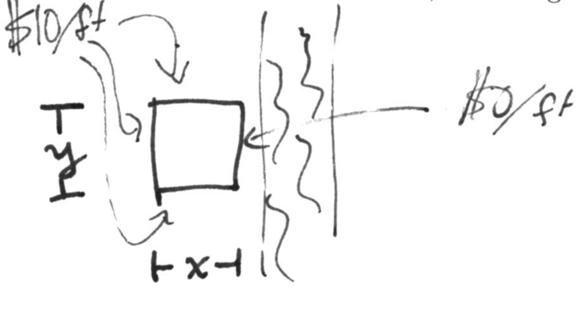
absolute max at $x = 56.25$

value of absolute max is $f(56.25) =$

7. [12 points] You want to fence off a rectangular pasture for your sheep. There is a river which will form the east side of the garden. The fencing for the other three sides costs \$10 per foot.

Find the dimensions of the garden largest garden you can build for \$9,000.

You must **show all work**, including verifying that this area is maximized.



NOTE

$$\left.\begin{aligned} \text{cost} &= 10x + 10y + 10x \\ 9000 &= 20x + 10y \\ y &= 900 - 2x \end{aligned}\right\} 4\text{pt}$$

maximize area subject to constraint

$$\left.\begin{aligned} F &= x \cdot y \\ &= x(900 - 2x) \\ F(x) &= 900x - 2x^2 \end{aligned}\right\} 4\text{pt} \qquad y = 900 - 2x$$

$$F'(x) = 900 - 4x$$

critical # at $F'(x) = 0$ when $900 = 4x$

$x = 225$

F	↗		↘
		225	
F′	(+)		(−)

2pt

Area maximized when

$x = 225$ ← 1pt

and

$$y = 900 - (225)(2)$$
$$= 900 - 450$$
$$= 450$$

1pt

8. (a) [6 points] Compute the general antiderivative for $f(x) = \dfrac{x+1}{x}$

$$f(x) = \frac{x}{x} + \frac{1}{x} = 1 + \frac{1}{x}$$ 2pt

$$F(x) = x + \ln|x| + C$$ 4pt

(b) [6 points] Suppose that $f'(x) = 8x^4 - 3e^x$, and that $f(0) = 5$. Find a formula for $f(x)$.

$$f(x) = \frac{8x^5}{5} - 3 \cdot e^x + C$$

$$f(0) = 5 = 0 - 3 \cdot e^0 + C$$

$$5 = -3 + C$$

$$C = 8$$

$$\boxed{f(x) = \frac{8}{5}x^5 - 3e^x + 8} \Leftarrow$$

9. (a) [6 points] Compute $\int_0^2 \left[e^x - 14x\right] dx$

$$= \left[e^x - \frac{14x^2}{2}\right]_0^2$$

$$= \left[e^x - 7x^2\right]_0^2 \qquad 3\text{pt}$$

$$= \left(e^2 - 7\cdot 4\right) - \left(e^0 - 7\cdot 0^2\right)$$

$$= e^2 - 28 - 1 + 0$$

$$= e^2 - 29 \qquad 3\text{pt}$$

(b) [6 points] Compute $\int_0^2 (x+5)(x-2)\, dx$

$$= \int_0^2 \left[x^2 + 3x - 10\right] dx \qquad 2\text{pt}$$

$$= \left[\frac{x^3}{3} + \frac{3x^2}{2} - 10x\right]_0^2 \qquad 2\text{pt}$$

$$= \left(\frac{8}{3} + \frac{12}{2} - 20\right) - \left(\frac{0}{3} + \frac{3\cdot 0}{2} - 10\cdot 0\right)$$

$$= \frac{8}{3} + 6 - 20$$

$$= \frac{8}{3} - 14 \qquad 2\text{pt}$$

1210

10. (a) [6 points] Compute $\int_0^1 \sqrt{1-x}\,dx$

$$\left(\begin{array}{l|l} u = 1-x & x=0 \Rightarrow u = 1-0 = 1 \\ \frac{du}{dx} = -1 & x = 1 \Rightarrow u = 1-1 = 0 \\ -du = dx & \end{array} \right)$$

2pt

$$= \int_1^0 u^{\frac{1}{2}} \cdot (-du)$$

2 options

$$= -\int_1^0 u^{\frac{1}{2}}\,du = \int_0^1 u^{\frac{1}{2}}\,du$$

2pt

$$= \left(\frac{-u^{\frac{3}{2}}}{\frac{3}{2}} \right)_1^0 = \left(\frac{-0}{\frac{3}{2}} - \frac{-1}{\frac{3}{2}} \right)$$

$$= \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

2pt

$$= \left[\frac{u^{3/2}}{3/2} \right]_0^1$$

$$= \frac{1}{\frac{3}{2}} - \frac{0}{\frac{3}{2}}$$

$$= \frac{2}{3}$$

(b) [6 points] Compute $\int \frac{x}{x^2+5}\,dx$ $\quad \frac{du}{2}$

$$\left(\begin{array}{l} u = x^2+5 \\ \frac{du}{dx} = 2x \\ \frac{du}{2} = x\,dx \end{array} \right)$$

3pt

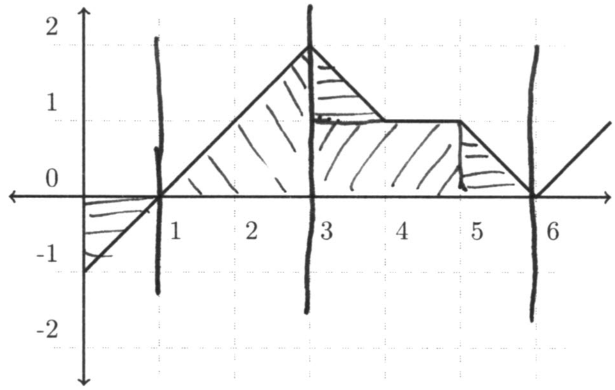
$$= \int \frac{1}{u} \cdot \frac{du}{2} = \frac{1}{2} \ln|u| + C$$

3pt

$$= \boxed{\frac{1}{2} \cdot \ln|x^2+5| + C}$$

1721

11. [8 points] Suppose that the function $f(x)$ is given by the following graph.



Let $A(x) = \int_0^x f(t)\,dt$. Compute the following

(2pt each)

(a) $A(1)$ $= -\frac{1}{2}$

(b) $A(3)$ $= -\frac{1}{2} + \frac{1}{2}\cdot 2\cdot 2 = -\frac{1}{2} + 2 = 1.5$

(c) $A(6) = -\frac{1}{2} + \frac{1}{2}\cdot 2\cdot 2 + \frac{1}{2} + 2\cdot 1 + \frac{1}{2}$

$= -\frac{1}{2} + 2 + \frac{1}{2} + 2 + \frac{1}{2} = 4.5$

(d) $A'(3)$ $= f(3) = 2$

$$\boxed{A'(x) = \frac{d}{dx}\left[\int_0^x f(t)\,dt\right] = f(x)}$$

1724

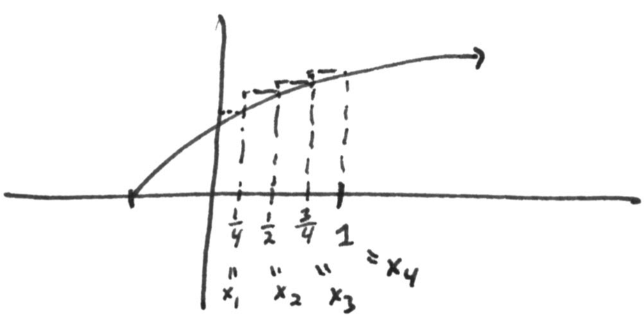
12. [12 points] Use the following Reimann Sums to approximate the integral $\int_a^b \sqrt{x+1}\,dx$

(a) Express the integral $\displaystyle\int_0^1 \sqrt{x+1}\,dx$ as the limit of its Right Reimann Sums.

$$\int_0^1 \sqrt{x+1}\,dx = \lim_{n\to\infty}\left[\sum_{i=1}^{n}\left(\sqrt{x_i+1}\cdot\Delta x\right)\right]$$

limit 1pt
Sum 1pt
rest 2pt

(b) Sketch a picture of the Right Sum approximation for $\displaystyle\int_0^1 \sqrt{x+1}\,dx$ when $n = 4$.



$$\Delta x = \frac{1-0}{4} = \frac{1}{4}$$

$x_1 = .25$

$x_2 = .5$

$x_3 = .75$

$x_4 = 1$

correct fn: 1pt
4 slices 1pt
Right rectangular 2pt

(c) Write out the Right Sum approximation for $\displaystyle\int_0^1 \sqrt{x+1}\,dx$ when $n = 4$. You must write out all numbers (endpoints and widths), but you do not need to simplify.

$$R_4 = \left(\sqrt{.25+1}\right)\cdot(.25) + \sqrt{.5+1}\,(.25) + \sqrt{.75+1}\cdot(.25) + \sqrt{1+1}\cdot(.25)$$

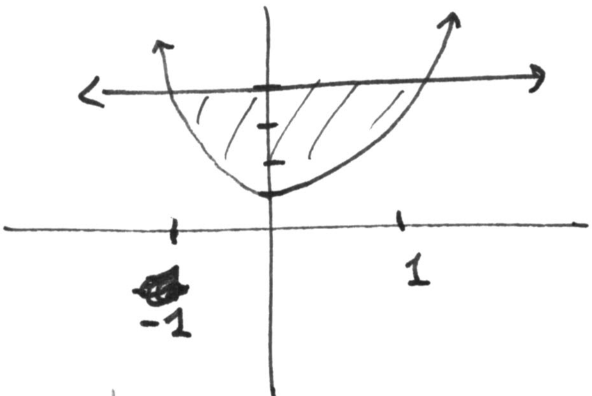
$$= (.25)\left(\sqrt{1.25} + \sqrt{1.5} + \sqrt{1.75} + \sqrt{2}\right)$$

$$\approx 1.27$$

correct formula: 2pt

correct endpoints & Δx: 2pt

13. [10 points] Find the area enclosed between $y = 4$ and $y = 3x^2 + 1$.



2pt

intersect when

$$4 = 3x^2 + 1$$
$$3 = 3x^2$$
$$1 = x^2$$
$$x = \pm 1$$

3pt

$$\text{area enclosed} = \int_{-1}^{1} \left[4 - (3x^2 + 1)\right] dx$$

3pt

$$= \int_{-1}^{1} \left[3 - 3x^2\right] dx$$

$$= \left[3x - x^3\right]_{-1}^{1}$$

$$= \left[3 \cdot 1 - (1)^3\right] - \left[3 \cdot (-1) - (-1)^3\right]$$

$$= [3 - 1] - [-3 + 1]$$

$$= 2 - (-2)$$

$$= 4$$

2pt