Instructions:

- This exam contains 10 pages. When we begin, check you have *one* of each page.
- You will have 75 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.** In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:

By writing my name below, I agree that all the work which appears on this exam is entirely my own.

I will not look at other peoples' work, and I will not communicate with anyone else about the exam.

I will not use any calculators, notes, etc.

I understand that violating the above carries *serious consequences*, both moral and academic.

Printed Name: Key Signature: ____________

Section: ____________

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	10	15	15	10	10	10	10	10	100
Score:										

1. [10 points] Elasticity

(a) Suppose that the price-demand function for miniature placards is given by $x(p) = 100 - 5p^2$. Find the elasticity of demand function $E(p)$.

$$E(p) = \frac{-p}{x} \cdot \frac{dx}{dp} \qquad \frac{dx}{dp} = -10p$$

$$= \frac{(-p)\cdot(-10p)}{100-5p^2}$$

2 pt $\boxed{E(p) = \frac{10p^2}{100-5p^2}}$

(a) ____________

(b) Find the price where miniature placards are unit elastic.

$$E(p) = 1 \iff \frac{10p^2}{100-5p^2} = 1$$ 2 pt

$$\iff 10p^2 = 100 - 5p^2$$ (price always ≥ 0)

$$15p^2 = 100$$

2 pt $$p = \pm\sqrt{\frac{100}{15}}$$

$\approx \$2.58$

(b) $\sqrt{\frac{100}{15}}$

(c) If the current price is \$2, what is the current elasticity?
How will a 2% increase in price impact demand?

$$E(2) = \frac{10\cdot 2^2}{100-5\cdot 2^2} = \frac{40}{100-20} = \frac{40}{80} = \frac{1}{2}$$ 2 pt

$$\left(\text{decrease in demand}\right) = E(2)\left(\%\text{ inc. in price}\right) = \frac{1}{2}\cdot 2\% = 1\%$$ 2 pt

(c) $\boxed{\text{1\% decrease in demand}}$

1203

2. [10 points] Let $f(x) = x^4 - 2x^3 + 6$

(a) Find the intervals where f is increasing and decreasing

4 pt

$$f'(x) = 4x^3 - 6x^2$$
$$= 2x^2(2x - 3)$$

Crit #'s when $f'(x)$ DNE or $f'(x) = 0$ when $x = 0$ or $x = \frac{3}{2}$

f

f' $\quad (-)^2(-) \quad 0 \quad (+)^2(-) \quad \frac{3}{2} \quad (+)^2(+)$

$(+)(-) \quad (-) \quad (+)$

$(-)$

increasing on $(\frac{3}{2}, \infty)$

decreasing on $(-\infty, 0) \cup (0, \frac{3}{2})$

(b) Find the intervals where f is concave up and concave down

3 pt

$$f''(x) = 12x^2 - 12x = 12x(x - 1)$$

f

f'' $\quad (-)(-) \quad 0 \quad (+)(-) \quad 1 \quad (+)(+)$

$(+) \quad (-) \quad (+)$

Concave up on $(-\infty, 0) \cup (1, \infty)$

concave down on $(0, 1)$

(c) Find the numbers x where the are local maxima and minima.
Write DNE if no such point exists.

2 pt

No local max.

local min at $\frac{3}{2}$

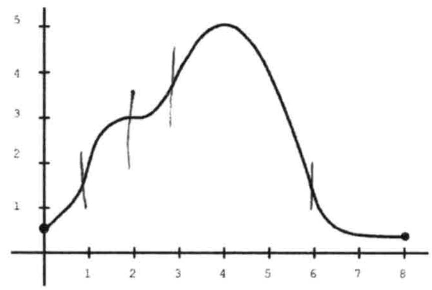
(d) Find the numbers x where the is a point of inflection. Write DNE if no such point exists.

1 pt

inflection at 0, 1.

3. [15 points] Let $f(x)$ be the function graphed below.

1pt per interval (rough endpts)
1pt - all endpoints exact



(a) Find the intervals where f is increasing and decreasing

inc on $(0, 4)$

dec on $(4, 8)$

(b) Find the intervals where f is concave up and concave down

CU: $(0,1) \cup (2,3) \cup (6,8)$

CD: $(1,2) \cup (3,6)$

(c) Find the numbers x where the are local maxima and minima

local max at 4

local min at 0 & 8

(d) Find the numbers x where the is a point of inflection

inflection at

1, 2, 3, 6.

1212

4. (a) [5 points] Compute

$$\lim_{x\to\infty} \frac{2x+3x^2}{5x^2+2}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^2}\left(\frac{2}{x}+3\right)}{\cancel{x^2}\left(5+\frac{2}{x^2}\right)} \qquad \frac{2}{x}\to 0,\ \frac{2}{x^2}\to 0$$

2 pt

$$= \boxed{\frac{3}{5}}$$

3pt

(b) [5 points] Compute

$$\lim_{x\to\infty} \frac{100x+900}{x^2-4}$$

$$= \lim_{x\to\infty} \frac{x\left(100+\frac{900}{x}\right)}{x^2\left(1-\frac{4}{x^2}\right)} \qquad \frac{900}{x}\to 0,\ \frac{4}{x^2}\to 0$$

2pt

$$= \lim_{x\to\infty} \frac{1}{x}\cdot\frac{100}{1} \quad \boxed{=0}$$

3pt

(c) [5 points] Compute

$$\lim_{x\to-\infty} \frac{x^4-7x^2}{x^3+2}$$

$$= \lim_{x\to-\infty} \frac{x^4\left(1-\frac{7}{x^2}\right)}{x^3\left(1+\frac{2}{x^3}\right)} \qquad \frac{7}{x^2}\to 0,\ \frac{2}{x^3}\to 0$$

2 pt

$$= \lim_{x\to-\infty} \frac{x}{1}\cdot\left(\frac{1}{1}\right) = \boxed{-\infty}$$

3 pt

1214

5. 10 points] Find the *absolute maximum* **and** the *absolute minimum* achieved by

$$f(x) = \ln(x) + \frac{2}{x} \text{ on the interval } [1, 100].$$

Find exact numbers using calculus, and use your calculator to give a decimal approximation.

3pt

$$f'(x) = \frac{1}{x} - \frac{2}{x^2}$$

3pt

critical #'s

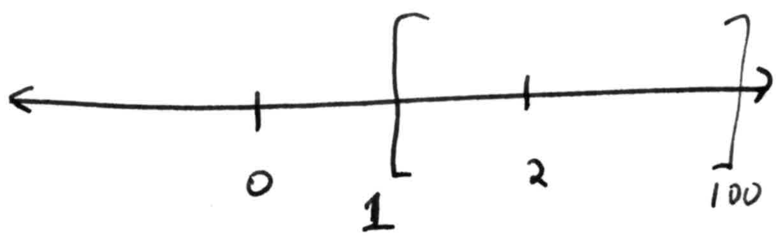
$f'(x)$ DNE when $x = 0$

$f'(x) = 0$ when $0 = \frac{1}{x} - \frac{2}{x^2}$

when $\frac{1}{x} = \frac{2}{x^2}$

$x^2 = 2x$

$x = 2$



Check end & critical points

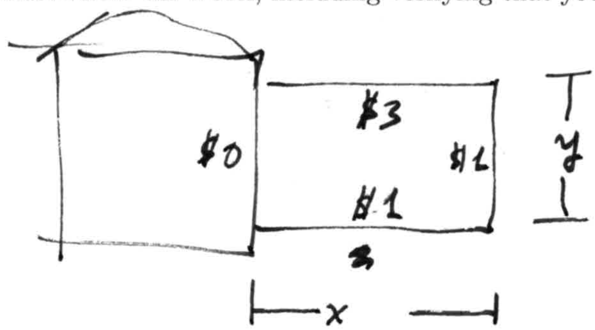
$f(1) = \ln(1) + \frac{2}{1} = 2$

abs min on [1,100] $\Rightarrow$ $f(2) = \ln(2) + \frac{2}{2} \approx 1.693$ 2pt

abs max on [1,100]. $\Rightarrow$ $f(100) = \ln(100) + \frac{2}{100} \approx 4.625$ 2pt

6. [10 points] You want to turn part of your yard into a dog park. For zoning reasons, you want the park to be exactly 100 ft^2. Suppose that your house will form the west boundary of the yard, that the fence for the north boundary will cost \$3 per foot and that the other two sides will cost \$1 per foot. What are the dimensions of the yard with the cheapest surrounding fence?

You must **show all work**, including verifying that your answer gives the minimum cost.



minimize:

2pt Cost = $0y + 1\cdot y + 3x + 1\cdot x$

$C = y + 4x$

constraint

$xy = 100$

$y = \frac{100}{x}$ 2pt

2pt $C(x) = \frac{100}{x} + 4x$

2pt $C'(x) = \frac{-100}{x^2} + 4$

2pt Crit #'s: $C'(x)$ DNE when $x=0$

$C'(x) = 0 \iff \frac{100}{x^2} = 4$

$4x^2 = 100$

$x^2 = 25$

$x = \pm 5$

but $x \geq 0$ in this problem

0 (−) 5 (+)

$C = 20 + 4\cdot 5$
$= 40$

Cost has abs. min when $x=5$ 2pt

$y = \frac{100}{5} = 20$

1220

7. [10 points] Suppose that the unit price function for metallic sculptures is $p = 905 - x^4$, and that the cost of each metallic sculpture is \$500.

(a) Find revenue, cost, and profit as functions of the number of units sold.

$$R(x) = 905x - x^5$$

$$C(x) = 500x$$

$$P(x) = (905x - x^5) - (500x)$$

2pt

$$= 405x - x^5$$

(b) Find the quantity you should sell to make the maximum *profit.*

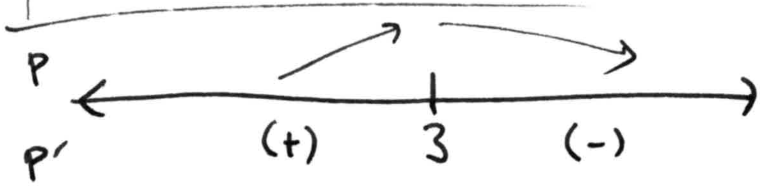
2pt $P'(x) = 405 - 5x^4$

2pt

$P'(x) = 0$ when $405 = 5x^4$

when $81 = x^4$

$x = 3$



$\boxed{\text{Profit max'd when } x = 3}$

2pt

(c) What is the maximum profit?

2pt $\boxed{P(3) = \$972}$

1223

8. (a) [5 points] Compute $\int \frac{x^3+2x}{x^2}\,dx$

$$= \int \left(\frac{x^3}{x^2} + \frac{2x}{x^2} \right) dx$$

2pt

$$= \int x\,dx + 2\cdot\int \frac{1}{x}\,dx$$

$$\boxed{= \frac{x^2}{2} + \ln|x| + C}$$

1 pt each

(b) [5 points] Compute $\int \frac{x}{\underbrace{x^2+2}_{u}}\,dx$ $\frac{du}{2}$

for all integrals

-1pt if no $|\cdot|$ in ln
-1pt if no +C

$$u = x^2+2$$

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2} = x\,dx$$

2 pt

$$= \int \frac{1}{u}\cdot\frac{du}{2} = \frac{1}{2}\ln|u| + C$$

1 pt

$$\boxed{= \frac{1}{2}\cdot\ln|x^2+2| + C}$$

2 pt

9. (a) [5 points] Compute $\displaystyle\int \frac{6(\overset{u}{\ln(x)})^2}{x}\,dx.$ du

$u = \ln(x)$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}dx$

← 3pt

$= \int 6u^2\,du$

$= \frac{6u^3}{3} + C = 2u^3 + C$

2pt

$\boxed{= 2(\ln(x))^3 + C}$

(b) [5 points] Suppose that $f'(x) = 8x^3 - 16x + 4$, and that $f(1) = 2$. Find $f(x)$.

$f(x) = 8\frac{x^4}{4} - \frac{16x^2}{2} + 4x + C$

2pt $f(x) = 2x^4 - 8x^2 + 4x + C$

2pt

$f(1) = 2 = 2\cdot1^4 - 8\cdot1^2 + 4\cdot1 + C$

$2 = 2 - 8 + 4 + C$

$C = 4$

$\Rightarrow \boxed{f(x) = 2x^4 - 8x^2 + 4x + 4}$ 1pt

1227