Instructions:

- This exam contains 10 pages. When we begin, check you have *one* of each page.
- You will have 75 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.**
  In order to receive full credit, solutions must be complete, logical, and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:

By writing my name below, I agree that all the work
which appears on this exam is entirely my own.

I will not look at other peoples' work,
and I will not communicate with anyone else about the exam.

I will not use any calculators, notes, etc.

I understand that violating the above carries *serious consequences*,
both moral and academic.

Printed Name: Key Signature: ____________

Section: ____________

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	8	8	15	12	12	15	10	10	10	100
Score:										

1. (a) [4 points] Simplify completely:

$$\left(\frac{2x^3y^{-1}}{y}\right)^3$$

$$= \left(\frac{2x^3}{y^2}\right)^3 = \frac{2^3 x^{3\cdot 3}}{y^{2\cdot 3}} = \boxed{\frac{8 \cdot x^9}{y^6}}$$

2pt 2pt

(a) ____________

(b) [4 points] Simplify completely:

$$e^{t\ln(2)}$$

$$= \left(e^{\ln(2)}\right)^t = 2^t$$

4pt

(b) ____________

2. (a) [4 points] Solve this equation for x:

$$2e^{3x} + 1 = 13$$

$$\frac{2e^{3x}}{2} = \frac{12}{2}$$

$$e^{3x} = 6$$

$$\ln(e^{3x}) = \ln(6)$$ 2pt

$$3x = \ln(6)$$

$$x = \frac{\ln(6)}{3}$$ 2pt

(a) ____________

(b) [4 points] Let $f(x) = 2\boxed{x}^2 - \boxed{x} - 1$ and $g(x) = x^2 - 1$.
Write down $f\big(g(x)\big)$ and simplify where possible.

$$f\big(g(x)\big) = f\left(\boxed{x^2-1}\right) = 2(x^2-1)^2 - (x^2-1) - 1$$ 2pt

$$= 2(x^4 - 2x^2 + 1) - x^2 \cancel{+1} \cancel{-1}$$

$$= 2x^4 - 4x^2 + 2 - x^2$$

$$= 2x^4 - 5x^2 + 2$$ 2pt

(b) ____________

$$F(t) = P \cdot \left(1 + \frac{r}{m}\right)^{mt}$$

3. [[redacted]] Suppose you make an investment at $\underbrace{1\% \text{ interest}}_{}$ (r), $\underbrace{\text{compounded twice each year}}_{}$ (m).

[5 points] (a) If your initial investment is \$5,000 (P), give the formula for the balance after t years (F(t)). Use this to find the balance after 12 years.

$$F(t) = 5000 \cdot \left(1 + \frac{0.01}{2}\right)^{2t}$$ 2pt

$$F(t) = 5000\,(1.005)^{2t}$$

after 12 yrs, the balance is

$$F(12) = 5000\,(1.005)^{2 \cdot 12}$$

$$= \$5635.80$$ 3pt

[5 points] (b) *How much* must your initial investment (P) be (at 1% interest compounded twice per year) to have a balance of \$10,000 (F) at the end of $\boxed{\text{12 years?}}$ (t)

want P such that

$$10{,}000 = P \cdot \left(1 + \frac{0.01}{2}\right)^{2 \cdot 12}$$ 2pt

$$10{,}000 = P\,(1.005)^{24}$$

$$P = \frac{10000}{(1.005)^{24}} = \boxed{\$8{,}871.86}$$ 3pt

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4. [redacted] *How long* (t) would it take an initial investment of \$5,000 (P) to grow to \$10,000 (F)?

[5 points]

want t such that

$$\frac{10{,}000}{5000} = \frac{5000\left(1 + \frac{0.01}{2}\right)^{2t}}{5000}$$

$$2 = (1.005)^{2t}$$ ← 2pt

$$\ln(2) = \ln\left((1.005)^{2t}\right) = 2t \cdot \ln(1.005)$$

$$t = \frac{\ln(2)}{2 \cdot \ln(1.005)} = 69.5 \text{ years}$$ 3pt

20:18

5.4 [12 points] Suppose you want to open an on-demand 3D printing business.

(a) You top choice printer has a fixed cost of $1,300 and the plastic costs $2 per unit produced. Write an equation for $C(x)$, the cost of producing x units.

$$C(x) = 1300 + 2x$$

3 pt

(b) Suppose the price function is given by price per unit $p = -0.01x + 10$. Write down an equation for $R(x)$, the revenue from selling x units.

$$R(x) = (\#\text{ units})(\text{price/unit})$$

$$= x(-0.01x + 10)$$

$$R(x) = -0.01x^2 + 10x$$

3pt

(c) Find the quantities where you break even.

$$R(x) = C(x)$$

$$\Leftrightarrow$$

4pt

$$1300 + 2x = -0.01x^2 + 10x$$

$$\Leftrightarrow$$

$$0.01x^2 - 8x + 1300 = 0$$

$$\Leftrightarrow$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{8 \pm \sqrt{8^2 - 4(0.01)(1300)}}{2 \cdot (0.01)} \begin{matrix} \nearrow 573.2 \\ \searrow 226.8 \end{matrix}$$

(d) How many units should you plan to make each year?

1pt you should make between 227 and 573 units

1pt you have maximum profit at

$$x = \frac{-b}{2a} = \frac{-(-8)}{2(0.01)} = 400 \text{ units}$$

2022

6. (a) [4 points] Write down the limit definition of the derivative of the function $f(x)$.

$$f'(x) =$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

(b) [4 points] Explain the graphical meaning of $f'(a)$ with words and with a sketch.

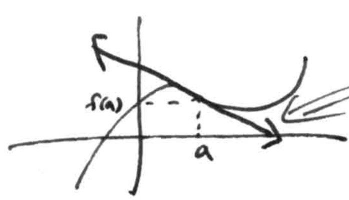
2pt

the instantaneous rate of change of $f(x)$ at a

OR

the slope of $f(x)$ at a

2pt



the slope of this line equals $f'(a)$

(c) [4 points] Let $f(x) = x^2 + 1$. Find $f'(1)$ using the limit definition of the derivative.

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$f(1+h) = (1+h)^2 + 1$

$= 1^2 + 2h + h^2 + 1$

$f(1) = 1^2 + 1$

2pt

$$= \lim_{h\to 0} \frac{(\cancel{1} + 2h + h^2 + \cancel{1}) - (\cancel{1} + \cancel{1})}{h}$$

$$= \lim_{h\to 0} \frac{2h + h^2}{h} = \lim_{h\to 0} [2 + h] = 2$$

2pt

2025

7. From this question onward, you may use the derivative rules. Show **major** steps for credit.

(a) [5 points] Let $f(x) = 3\sqrt{x} - \frac{3}{x} + 37$. Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[3x^{\frac{1}{2}} - 3x^{-1} + 37\right]$$

$$= 3\cdot\frac{1}{2}x^{-\frac{1}{2}} - 3(-1)\,x^{-2} + 0$$

$$= \frac{3}{2\sqrt{x}} + \frac{3}{x^2}$$

5pt

(b) [5 points] Let $y = x^4 e^x$. Find $\frac{dy}{dx}$.

product rule

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^4\cdot e^x\right] = x^4\cdot\frac{d}{dx}\left[e^x\right] + e^x\cdot\frac{d}{dx}\left[x^4\right]$$ 2pt

$$= x^4\cdot e^x + e^x\cdot 4x^3$$

$$= e^x\left(x^4 + 4x^3\right)$$

$$\boxed{= e^x\cdot x^3\left(x + 4\right)}$$

3pt

(c) [5 points] Let $f(x) = \frac{\ln(x)}{1-3x}$. Find $f'(x)$.

$$\left(\frac{t}{b}\right)' = \frac{bt' - tb'}{b^2}$$

$$f'(x) = \frac{d}{dx}\left[\frac{\ln(x)}{1-3x}\right]$$

$$= \frac{(1-3x)\frac{d}{dx}\left[\ln(x)\right] - \ln(x)\cdot\frac{d}{dx}\left[1-3x\right]}{(1-3x)^2}$$ 2pt

$$= \frac{(1-3x)\cdot\frac{1}{x} - \ln(x)\cdot(-3)}{(1-3x)^2} = \cdots$$ 3pt

2028

8. (a) [5 points] Let $f(x) = \sqrt{1+6x^2}$. Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left[(1+6x^2)^{\frac{1}{2}}\right]$$

$$\boxed{\begin{array}{l} \underline{\text{chain rule}} \\ g(u) = u^{\frac{1}{2}} \Rightarrow g'(u) = \frac{1}{2}u^{-\frac{1}{2}} \\ f(x) = 1+6x^2 \Rightarrow f'(x) = 12x \end{array}}$$

2pt

$$= \frac{1}{2}(1+6x^2)^{-\frac{1}{2}} \cdot 12x$$

$$= \frac{6x}{\sqrt{1+6x^2}}$$

3pt

(b) [5 points] Let $f(x) = \ln(e^x - 7x)$. Find $f'(x)$

$$f'(x) = \frac{d}{dx}\left[\ln(e^x - 7x)\right]$$

$$\boxed{\begin{array}{l} g(u) = \ln(u) \Rightarrow g'(u) = \frac{1}{u} \\ f(x) = e^x - 7x \Rightarrow f'(x) = e^x - 7 \end{array}}$$

2pt

$$= \frac{1}{e^x - 7x} \cdot (e^x - 7)$$

$$= \frac{e^x - 7}{e^x - 7x}$$

3pt

9. [10 points] Let $f(x)$ be the function

$$f(x) = 0.1x^2 - 10x + 1000$$

(a) Find the *slope* of the tangent line to $f(x)$ at $x = 10$.

$f'(x) = \frac{d}{dx}\left[0.1x^2 - 10x + 1000\right]$

$f'(x) = 0.2x - 10$ ← 2pt

Slope of tangent at 10 $= f'(10) = (0.2)(10) - 10$

$= 2 - 10$

$= -8$ ← 3pt

(b) Find the *equation* of the tangent line to $f(x)$ at $x = 10$.

$x_1 = 10$

$\Rightarrow y_1 = f(10) = 0.1(10)^2 - 10(10) + 1000$

$= 10 - 100 + 1000$

$= 10 + 900$

$y_1 = 910$

$m = f'(10) = -8$

$y = m(x - x_1) + y_1$ ← 5

$y = -8(x-10) + 910 = -8x + 990$

2035

10. [10 points] Suppose that the cost function for a certain product is $C(x) = x(\sqrt{x} + 1)$.

(a) Find the marginal cost.

$$C'(x) = \frac{d}{dx}\left(x \cdot \left(x^{\frac{1}{2}} + 1\right)\right)$$

$$= \frac{d}{dx}\left[x^{\frac{3}{2}} + x\right]$$

3 points

$$C'(x) = \frac{3}{2}x^{\frac{1}{2}} + 1 = \frac{3}{2}\sqrt{x} + 1$$

(b) Find $C'(16)$. What is the business meaning of this?

$$C'(16) = \frac{3}{2}\sqrt{16} + 1 = \frac{3}{2} \cdot 4 + 1$$

$$= 6 + 1 = 7$$

2pt

$7 = C'(16) \approx$ the cost of one more unit (the 17^{th} unit)

2pt

(c) Find the average rate of change of the cost over the interval $[16, 17]$. What is the business meaning of this?

$$\frac{C(17) - C(16)}{17 - 16} = \frac{17(\sqrt{17} + 1) - 16(\sqrt{16} + 1)}{1}$$

2pts

$$= 7.09$$

this is the actual cost of the 17^{th} unit.

1 pt.