Trigonometric Identity Review

Name: ______________________________ Section: ______________________________

E.g. Prove the following identity

$$\frac{\cos(\theta)}{1-\sin^2(\theta)} = \sec(\theta)$$

$$\frac{\cos(\theta)}{\boxed{1-\sin^2(\theta)}}$$

$$= \frac{\cos(\theta)}{\cos^2(\theta)}$$

$$= \frac{1}{\cos(\theta)}$$

$$= \sec(\theta)$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$-\sin^2(\theta) \qquad -\sin^2(\theta)$$

$$\boxed{\cos^2(\theta)} = \boxed{1-\sin^2(\theta)}$$

$$\frac{a}{a^2} = \frac{1}{a}$$

Trigonometric Identity Review

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E.g. Prove the following identity

$$(\cos(\theta) - \sin(\theta))^2 = 1 - \sin(2\theta)$$

$$(\cos(\theta) - \sin(\theta))^2 = (\cos(\theta) - \sin(\theta))(\cos(\theta) - \sin(\theta))$$

FOIL

$$= \cos(\theta)\cdot\cos(\theta) - \cos(\theta)\cdot\sin(\theta) - \sin(\theta)\cdot\cos(\theta) + \sin^2(\theta)$$

$$= \cos^2(\theta) - 2\cdot\cos(\theta)\cdot\sin(\theta) + \sin^2(\theta)$$

$$= \cos^2(\theta) + \sin^2(\theta) - 2\cdot\sin(\theta)\cdot\cos(\theta)$$

$$= 1 - 2\cdot\sin(\theta)\cdot\cos(\theta)$$

$$= 1 - \sin(2\theta)$$

Name: ____________________ Section: ____________________

E.g. Prove the following identity

$$\cos(2\theta)\sec^2(\theta) = 1 - \tan^2(\theta)$$

$$\cos(2\theta)\cdot\sec^2(\theta) = \cos(2\theta)\cdot\frac{1}{\cos^2(\theta)}$$

$$= \left(\cos^2(\theta) - \sin^2\theta\right)\cdot\frac{1}{\cos^2(\theta)}$$

$$= \frac{\cos^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= 1 - \tan^2\theta$$

Trigonometric Identity Review

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E.g. Prove the following identity

$$\frac{\csc(\theta)}{2\cos(\theta)} = \csc(2\theta)$$

$$\frac{\csc(\theta)}{2\cdot\cos(\theta)} = \frac{\frac{1}{\sin(\theta)}}{2\cdot\cos(\theta)}\cdot\frac{\sin(\theta)}{\sin(\theta)} \quad \text{trig defn}$$

$$= \frac{1}{2\cdot\sin(\theta)\cdot\cos(\theta)}$$

$$= \frac{1}{\sin(2\theta)} \quad \text{double } \measuredangle \text{ identity}$$

$$= \csc(2\theta)$$

Trigonometric Identity Review

Name: ____________________ Section: ____________________

E.g. Prove the following identity

$$(2\sin(\theta))^4 = 2\cos(4\theta) - 8\cos(2\theta) + 6$$

$$(ab)^4 = a^4 \cdot b^4$$

$$a^{rs} = (a^r)^s$$

$$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$$

$$(2\sin(\theta))^4 = 2^4 \cdot (\sin(\theta))^4$$

$$= 2^4 \cdot \left(\boxed{\sin^2(\theta)}\right)^2$$

$$= 2^4 \left(\boxed{\frac{1-\cos(2\theta)}{2}}\right)^2$$

$$= 2^4 \cdot \frac{(1-\cos(2\theta))^2}{2^2}$$

$$= 2^2 \cdot \left(1 - 2\cdot 1\cdot \cos(2\theta) + \cos^2(2\theta)\right)$$

$$= 2^2 \cdot \left(1 - 2\cdot\cos(2\theta) + \cos^2(\underbrace{\boxed{2\theta}}_{u})\right)$$

$$\cos^2(u) = \frac{1+\cos(2u)}{2}$$

$$= 2^2 \cdot \left(1 - 2\cdot\cos(2\theta) + \left(\frac{1+\cos(4\theta)}{2}\right)\right)$$

$$= 4 - 8\cos(2\theta) + 2\cancel{4}\left(\frac{(1+\cos 4\theta)}{\cancel{2}}\right)$$

$$= \underline{4} - 8\cos(2\theta) \underline{+2} + 2\cdot\cos(4\theta)$$

$$= 2\cdot\cos(4\theta) - 8\cdot\cos(2\theta) + 6$$