Instructions:

- This exam contains 13 pages. When we begin, check you have *one* of each page.
- You will have 75 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.**
  In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:
By writing my name below, I agree that all the work
which appears on this exam is entirely my own.
I will not look at other peoples' work,
and I will not communicate with anyone else about the exam.
I will not use any calculators, notes, etc.
I understand that violating the above carries *serious consequences*,
both moral and academic.

Printed Name: Key Signature: ____________

Section: ____________

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	9	10	10	10	5	10	10	8	10	8	10	100
Score:												

1. (a) [4 points] You have an analog clock with a 6" long hour hand. Find the angular and linear velocity at the end of the hour hand.

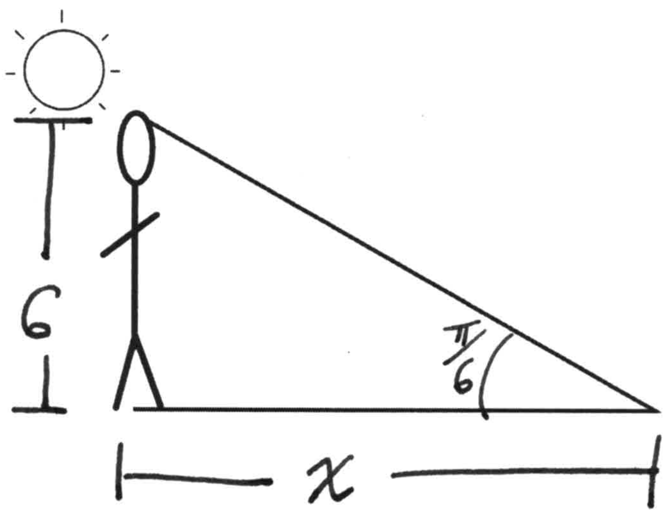
2pt

$$\text{angular velocity} = \frac{2\pi \text{ radian}}{12 \text{ hrs}} = \frac{2\pi}{12} \frac{\text{rad}}{\text{hr}} = \frac{\pi}{6} \text{ rad/hr}$$

2pt

$$\text{linear velocity} = \left(\begin{matrix}\text{angular}\\ \text{velocity}\end{matrix}\right) \cdot (\text{radius}) = \frac{\pi}{6} \cdot 6'' = \pi \text{ inches/hr}$$

(b) [5 points] A 6' tall person is standing outside on a sunny day. The sun makes an angle of $\frac{\pi}{6}$ radians with the ground. How long is the person's shadow?



2pt

$$\tan\left(\frac{\pi}{6}\right) = \frac{6}{x}$$

and

2pt

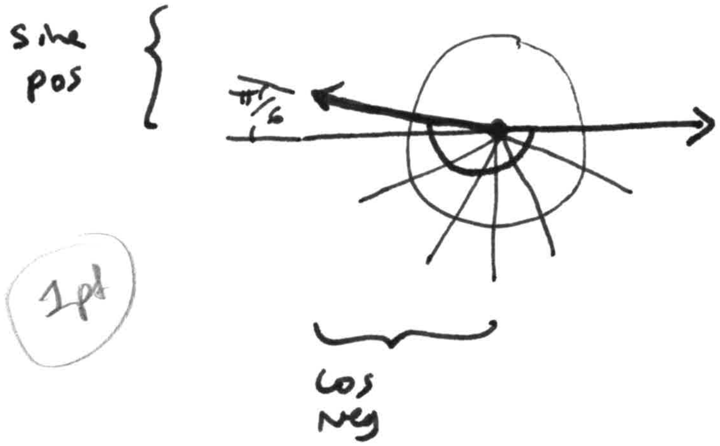
$$\tan\left(\frac{\pi}{6}\right) = \frac{\sin(\pi/6)}{\cos(\pi/6)} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{6}{x} = \frac{1}{\sqrt{3}}$$

$$\boxed{x = 6\sqrt{3} \text{ feet}}$$

1 pt

2. [10 points] Evaluate 6 trigonometric functions at the angle $\theta = \frac{-7\pi}{6}$



ref $\measuredangle = \pi/6$

$\sin(\pi/6) = 1/2$

$\cos(\pi/6) = \sqrt{3}/2$

1pt

2pt each

$\sin\left(\frac{-7\pi}{6}\right) = 1/2$

$\cos\left(\frac{-7\pi}{6}\right) = \frac{-\sqrt{3}}{2}$

$\tan\left(\frac{-7\pi}{6}\right) = \frac{1/\cancel{2}}{-\sqrt{3}/\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{2}} = \frac{1}{-\sqrt{3}}$

1 pt each

$\csc\left(\frac{-7\pi}{6}\right) = 2$

$\sec\left(\frac{-7\pi}{6}\right) = \frac{-2}{\sqrt{3}}$

$\cot\left(\frac{-7\pi}{6}\right) = -\sqrt{3}$

3. [10 points] Give *all solutions* to the following equation.

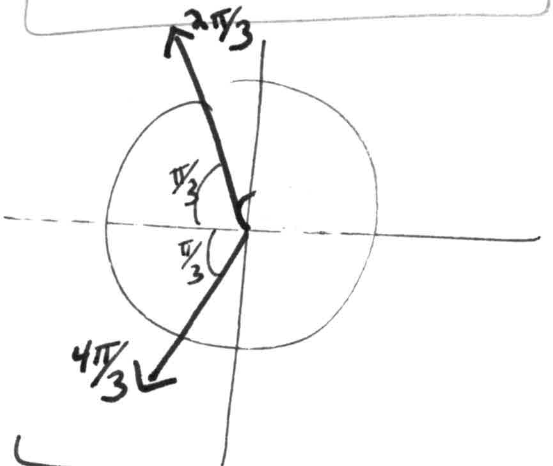
$$2\cos(t) + 1 = 0$$

(For full credit, you must write your answer without using inverse trig functions).

$$2 \cdot \cos(t) = -1$$

2pt $\boxed{\cos(t) = -\tfrac{1}{2}}$ $\Rightarrow$ (ref ∡) $= \pi/3$

2pt



Cos is **negative**

2pt for correct quadrant

All solutions are

1 pt ea for basic slns

$$t = \frac{2\pi}{3} + 2\pi \cdot n$$

and

$$t = \frac{4\pi}{3} + 2\pi \cdot n$$

for **ALL** integers n

2pt for $2\pi n$ to all basic slns

−2pts if you don't write what you mean

ex WRONG: $\cos\left(\frac{2\pi}{3}\right) + 2\pi n$

RIGHT $\frac{2\pi}{3} + 2\pi n$

max 6/10 if used inverse trig in answer

4. (a) [5 points] Suppose that θ is an unknown angle such that $\sin(\theta) = \dfrac{2}{3}$ and that $\cos(\theta)$ is negative. Find $\cos(\theta)$.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\left(\tfrac{2}{3}\right)^2 + \cos^2(\theta) = 1$$

2pt

$$\frac{4}{9} + \cos^2\theta = \frac{9}{9}$$

$$\cos^2\theta = \frac{5}{9}$$

$$|\cos\theta| = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$$

2pt

$\cos(\theta)$ neg

$$\Rightarrow \cos\theta = \frac{-\sqrt{5}}{3}$$

1 pt

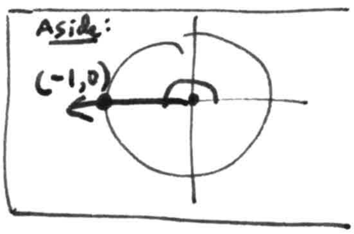
(b) [5 points] Suppose that u is a mystery angle with $\sin(u) = 2/3$ and $\cos(u) = \sqrt{5}/3$.

Find $\cos(u - \pi)$.

You must **show all work** for full credit.

$$\cos(u-\pi) = \cos(u)\cdot\cos(\pi) + \sin(u)\cdot\sin(\pi)$$

2pt



$$= \frac{\sqrt{5}}{3}\cdot(-1) + \frac{2}{3}\cdot 0$$

2pt

$$= \frac{-\sqrt{5}}{3}$$

1 pt

5. (a) [5 points] Suppose that u is a mystery angle with $\sin(u) = 2/3$ and $\cos(u) = \sqrt{5}/3$.

Find $\cos(2u)$.

You must **show all work** for full credit.

$$\cos(2u) = \cos^2(u) - \sin^2(u)$$

$$= \left(\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{2}{3}\right)^2$$

3 pt

$$= \frac{5}{9} - \frac{4}{9}$$

$$= \frac{1}{9}$$

2pt

6. [10 points] Use the half angle identity to compute the following.

$$\cos\left(\frac{3\pi}{8}\right)$$

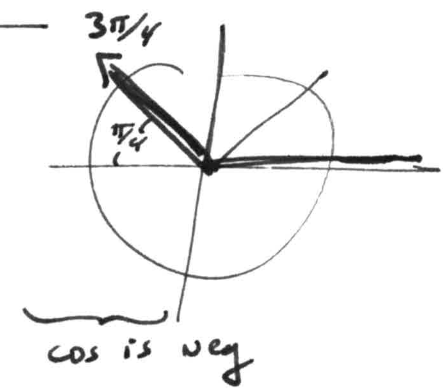
$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

$$\frac{u}{2} = \frac{3\pi}{8} \Rightarrow \quad u = \frac{3\pi}{4}$$ 2pt

$$\cos\left(\frac{3\pi}{8}\right) = \pm\sqrt{\frac{1+\cos\left(\frac{3\pi}{4}\right)}{2}}$$ 3pt

3pt

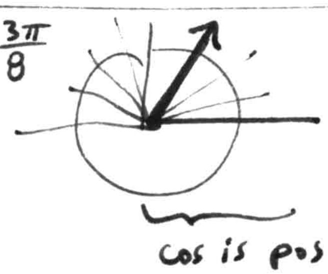
$$\cos(3\pi/4) = -\frac{1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$



$$= \pm\sqrt{\frac{1+\frac{-1}{\sqrt{2}}}{2}}$$

2pt

Graph $\frac{3\pi}{8}$



$$= + \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}$$

7. (a) [5 points] Prove the following trigonometric identity.

$$\frac{\tan(\theta)}{\sec^2(\theta)-1} = \cot(\theta)$$

You must show **all steps** for full credit.

$$\boxed{\begin{aligned} \tan^2\theta + 1 &= \sec^2\theta \\ \tan^2\theta &= \sec^2\theta - 1 \end{aligned}}$$

$$\frac{\tan\theta}{\sec^2\theta-1} = \frac{\tan\theta}{\tan^2\theta}$$

$$= \frac{1}{\tan\theta}$$

$$= \cot(\theta)$$

(b) [5 points] Prove the following trigonometric identity.

$$\sin(2\theta)\cdot(1+\tan^2(\theta)) = 2\tan(\theta)$$

You must show **all steps** for full credit.

1pt

$$\sin(2\theta)\cdot\left(1+\tan^2(\theta)\right)$$

$$= \boxed{\left(2\cdot\sin(\theta)\cdot\cos(\theta)\right)}\cdot\boxed{\left(\sec^2(\theta)\right)}$$

2pt

$$= 2\cdot\sin(\theta)\cdot\cos(\theta)\cdot\boxed{\frac{1}{\cos^2\theta}}$$

2pt

$$= 2\cdot\frac{\sin(\theta)}{\cos(\theta)}$$

$$\boxed{= 2\cdot\tan(\theta)}$$

full credit.

8. [8 points] Prove the following trigonometric identity.

$$\cos^4(\theta) = \frac{3}{8} + \frac{\cos(2\theta)}{2} + \frac{\cos(4\theta)}{8}$$

You must show **all steps** for full credit.

$$\cos^4(\theta) = \left(\cos^2(\theta) \right)^2$$

1 pt

$$= \left(\frac{1 + \cos(2\theta)}{2} \right)^2$$

2 pt

$$= \frac{(1 + \cos(2\theta))^2}{2^2}$$

$$\cos^2(u) = \frac{1 + \cos(2u)}{2}$$

$$= \frac{1 + 2\cdot\cos(2\theta) + \boxed{\cos^2(\underbrace{2\theta}_{u})}}{4}$$

2 pt

$$= \frac{1}{4} + \frac{2}{4}\cos(2\theta) + \frac{1}{4}\cdot\left(\boxed{\frac{1 + \cos(2\cdot 2\theta)}{2}} \right)$$

2 pt

$$= \frac{1}{4} + \frac{1}{2}\cdot\cos(2\theta) + \frac{1}{8}\left(1 + \cos(4\theta)\right)$$

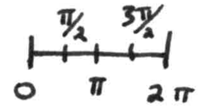
1 pt

$$= \frac{2}{8} + \frac{\cos(2\theta)}{2} + \frac{1}{8} + \frac{\cos(4\theta)}{8}$$

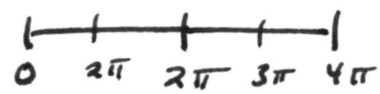
$$= \frac{3}{8} + \frac{\cos(2\theta)}{2} + \frac{\cos(4\theta)}{8}$$

9. Graph the function below, and give its period.

(a) [5 points] $g(x) = 2\cos\left(\frac{x}{2}\right)$



$\Rightarrow$ stretch to get



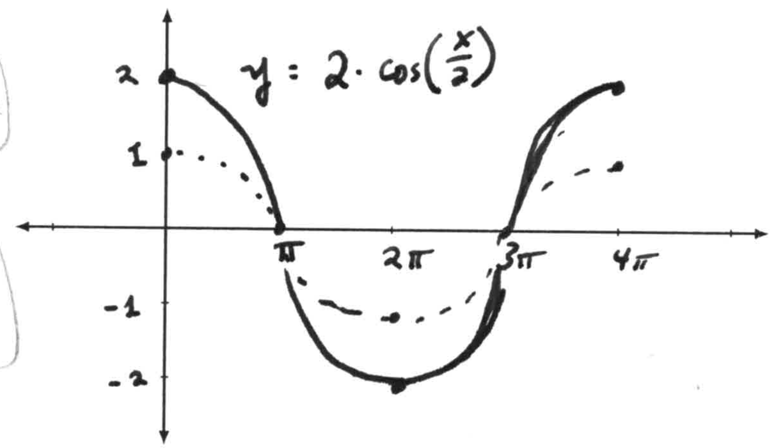
1 pt

Period: $\dfrac{2\pi}{\frac{1}{2}} = 4\pi$

Be sure you fill in the scale for the x and y axes.

2pt for work

2pt for graph & axis



cos(x)

Stretched horiz by factor of 2

&

stretched vertically by factor of 2

(b) [5 points] $f(x) = \sin\left(x - \frac{\pi}{2}\right) + 1$

sin(x)

Shifted RIGHT $\pi/2$

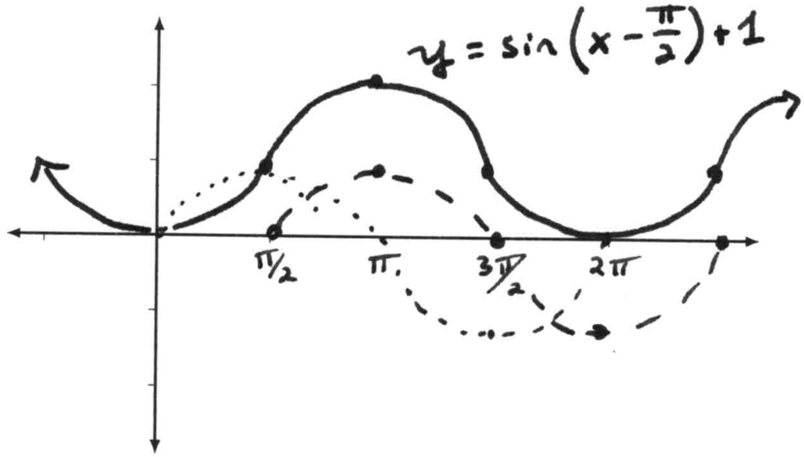
and

up 1

Same as (a)

Period: 2π

Be sure you fill in the scale for the x and y axes.

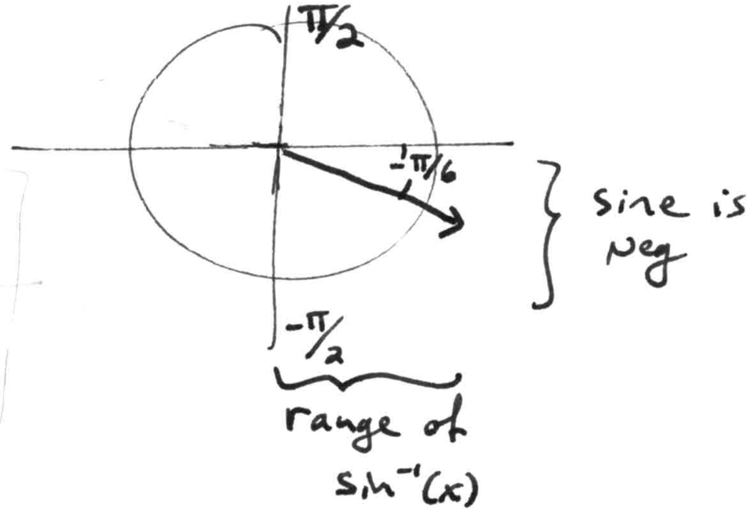


10. [8 points] Find the *exact* value of the following.

(a) $\sin^{-1}\left(-\frac{1}{2}\right)$

$= -\pi/6$

ref $\measuredangle = \pi/6$



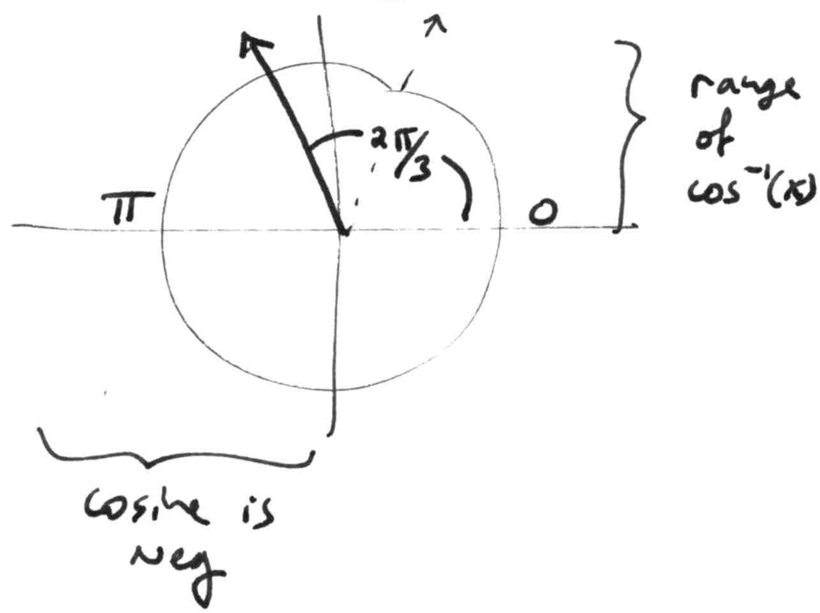
2pts sin (answer) does equal $\frac{-1}{2}$

for correct $\measuredangle$'s:
2pts ONLY ONE output which is in $[-\pi/2, \pi/2]$

(b) $\cos^{-1}\left(-\frac{1}{2}\right)$

$= 2\pi/3$

ref $\measuredangle = \pi/3$



2pts $\cos(\text{answer}) = \frac{-1}{2}$

for correct $\measuredangle$'s:
2pt ONLY ONE output which is IN $[0, \pi]$

11. [10 points] Use inverse trigonometric functions to find all the solutions to the equation

$$6\sin(x) - 3 = 2\sin(x)$$

$$-2\sin(x) \quad -2\sin(x)$$

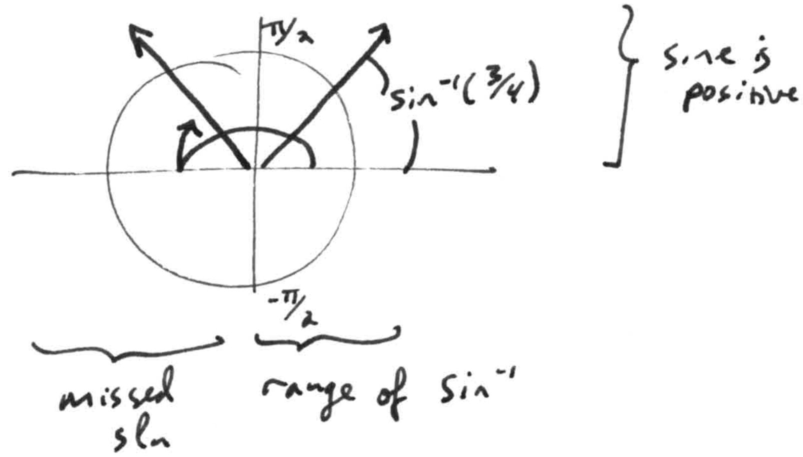
$$4\cdot\sin(x) - 3 = 0$$

$$4\cdot\sin(x) = 3$$

$$\sin(x) = 3/4$$

3pt

2pt **one** solution: $x = \sin^{-1}(3/4)$



3pt **missed** solution: $x = \pi - \sin^{-1}(3/4)$

2pt for $+2\pi\cdot n$

all solutions are

$$\boxed{\begin{array}{l} \sin^{-1}(3/4) + 2\pi\cdot n \\ \text{and} \\ \pi - \sin^{-1}(3/4) + 2\pi n \end{array} \Bigg\} \text{ for some integer } n}$$

Pythagorean Identities

- $\sin^2(x) + \cos^2(x) = 1$
- $\tan^2(x) + 1 = \sec^2(x)$
- $1 + \cot^2(x) = \csc^2(x)$

Sum and Difference of Angles

- $\sin(u \pm v) = \sin(u)\cos(v) \pm \cos(u)\sin(v)$
- $\cos(u \pm v) = \cos(u)\cos(v) \mp \sin(u)\sin(v)$

Double Angle Identities

- $\sin(2u) = 2\sin(u)\cos(u)$
- $\cos(2u) = \cos^2(u) - \sin^2(u)$
  - ▷ $\cos(2u) = 2\cos^2(u) - 1$
  - ▷ $\cos(2u) = 1 - 2\sin^2(u)$

Squared Trig Identities

- $\cos^2(u) = \dfrac{1 + \cos(2u)}{2}$
- $\sin^2(u) = \dfrac{1 - \cos(2u)}{2}$

Half Angle Identities

- $\cos(\frac{x}{2}) = \pm\sqrt{\dfrac{1 + \cos(x)}{2}}$
- $\sin(\frac{x}{2}) = \pm\sqrt{\dfrac{1 - \cos(x)}{2}}$