Instructions:

- This exam contains 13 pages. When we begin, check you have *one* of each page.
- You will have 75 minutes to complete the exam.
- Please **show all work**, and then **write your answer on the line provided.**
  In order to receive full credit, solutions must be complete, logical and understandable.
- Turn smart phones, cell phones, and other electronic devices off now!

Academic Honesty:

By writing my name below, I agree that all the work
which appears on this exam is entirely my own.

I will not look at other peoples' work,
and I will not communicate with anyone else about the exam.

I will not use any calculators, notes, etc.

I understand that violating the above carries *serious consequences*,
both moral and academic.

Printed Name: Key Signature: ______________

Section: ______________

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	10	9	8	5	10	8	10	10	8	14	8	100
Score:												

1. For each of the following equalities, assume that x and y stand for some real numbers, and that all denominators are non-zero.

   (a) [5 points] Is the statement $\frac{1}{y} + \frac{1}{x}$ always equal to $\frac{x+y}{xy}$? Give a proof or counterexample.

Yes

proof: $\frac{x}{x} \cdot \frac{1}{y} + \frac{1}{x} \cdot \frac{y}{y} = \frac{x}{xy} + \frac{y}{xy}$

$= \frac{x+y}{xy}$ ✓

2pts - answer

3 pts - proof

   (b) [5 points] Is the statement $\sqrt{x^2+4}$ always equal to $x+2$?
   Give a proof or counterexample.

No

counterexample: if $x = 2$,

$\sqrt{2^2+4} = \sqrt{8}$

But

$2+2 = 4$

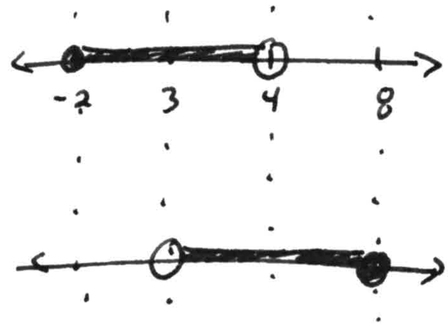
NOT equal.

2pts - answer

3pts - counterexample

2. (a) [4 points] Rewrite the following using interval notation:

1. $[-2, 4) \cup (3, 8]$



in <u>either</u>

$[-2, 8]$

2pt

2. $[-2, 4) \cap (3, 8]$

in <u>BOTH</u>

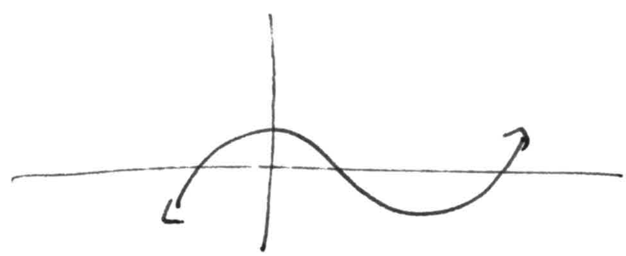
$(3, 4)$

2pt

(b) [5 points]

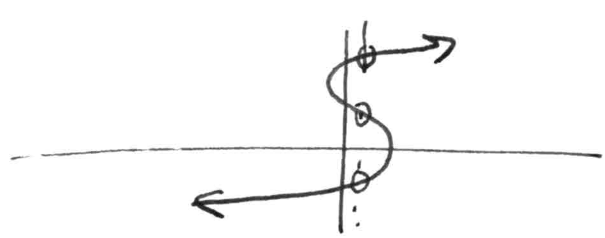
1. Sketch a graph that passes the vertical line test

2pt



2. Sketch a graph that fails the vertical line test

2pt



3. Which of the above two graphs defines a function? why?

1pt

<u>the first graph.</u>
Each input x can have <u>at most one</u> output y above/below it.

3. [8 points] Suppose you want to open an on-demand 3D printing business.

(a) You top choice printer has a fixed cost of \$1,300 and the plastic costs \$2 per unit produced. Write an equation for $C(x)$, the cost of producing x units.

$$C(x) = 1300 + 2x$$

2pt

(b) If each unit sells for \$12, give an equation for $R(x)$, the income from selling x units.

$$R(x) = 12x$$

2pt

(c) Find the quantities where you "break even".

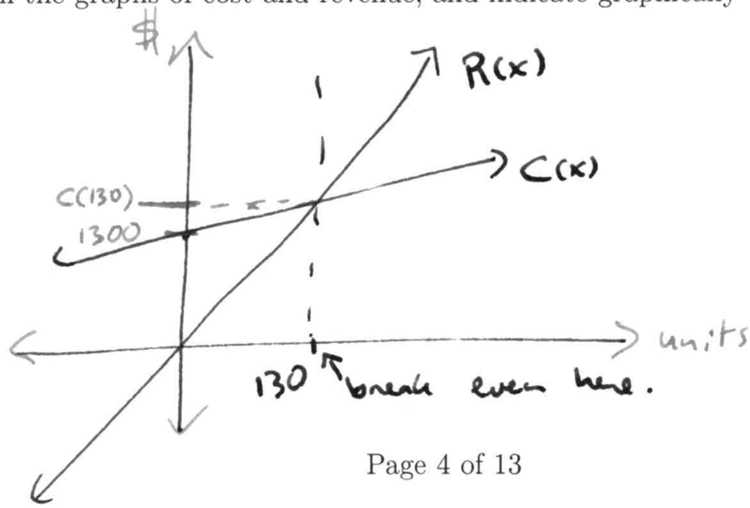
Break even when

$$C(x) = R(x)$$
$$1300 + 2x = 12x$$
$$1300 = 10x$$
$$x = 130$$

2pt

(d) Sketch the graphs of cost and revenue, and indicate graphically where you "break even"

2pt



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4. [5 points] Find the equation for a linear function going through $(3,6)$ and $(7,8)$. Write your answer both in point slope form and in slope intercept form.

$$y = m(x - x_1) + y_1$$

2pt

$$m = \frac{\text{rise}}{\text{run}} = \frac{8-6}{7-3} = \frac{2}{4} = \frac{1}{2}$$

3pt

$$f(x) = \frac{1}{2}(x-3) + 6 \quad \leftarrow \text{point slope form}$$

$$= \frac{1}{2}x - \frac{3}{2} + 6$$

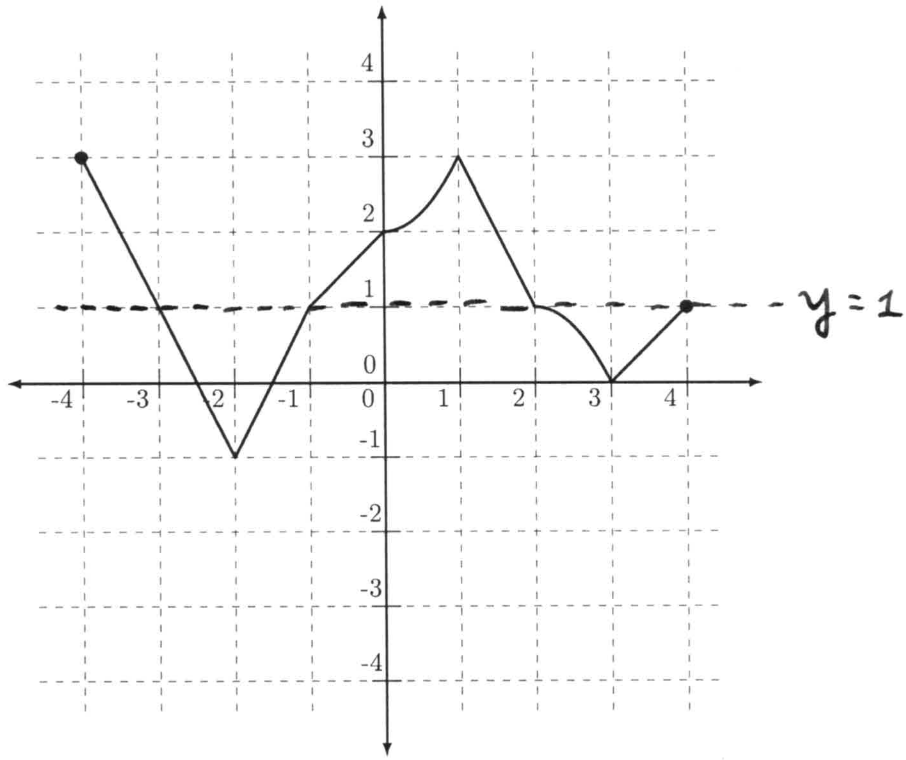
$$= \frac{1}{2}x - \frac{3}{2} + \frac{12}{2}$$

$$f(x) = \frac{1}{2}x + \frac{9}{2} \quad \longleftarrow \text{Slope intercept form}$$

=

7:01

5. [10 points] Suppose that $f(x)$ is defined using the following graph.



1 pt each

(a) Compute $f(0)$ and $f(3)$.

$f(0) = 2$

$f(3) = 0$

(b) Find the domain and range of f.

Domain = $[-4, 4]$

Range = $[-1, 3]$

(c) For what x is $f(x) = 1$?

when $x = -3, -1, 2, 4$

(d) For what x is $f(x) < 1$?

when x is in $(-3,-1) \cup (2,4)$

6. [8 points] (a) Complete the square to rewrite $f(x) = x^2 - 12x + 34$ in the form $a(x-h)^2 + k$.

$$f(x) = x^2 - 12x + \underline{36} - \underline{36} + 34$$ 2pt

$-\frac{12}{2} = -6$

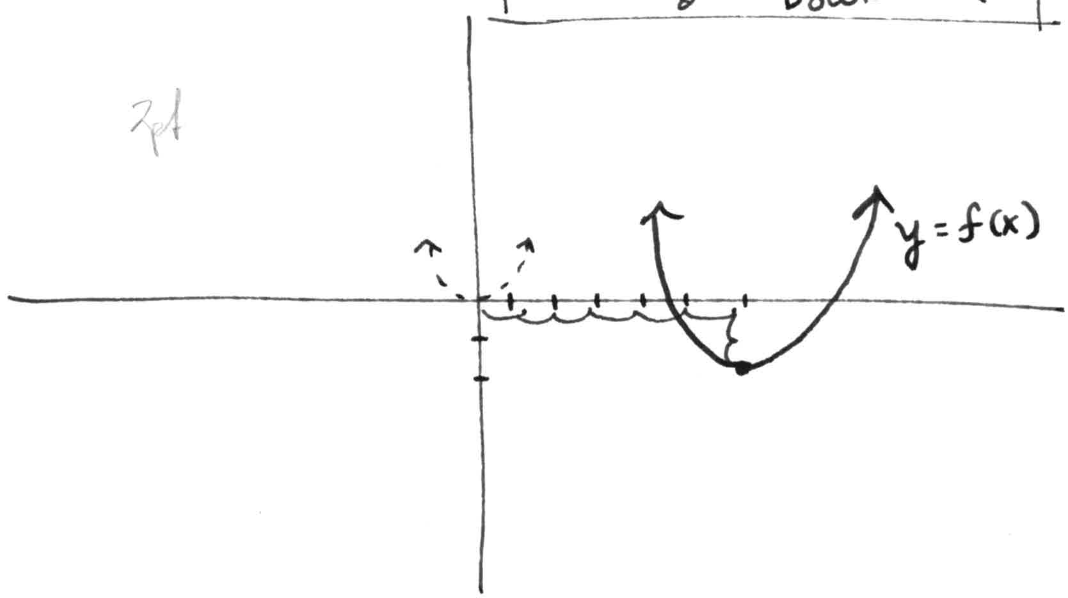
$$= (x-6)(x-6) - \underline{36} + 34$$ 2pt

$$= (x-6)^2 - 2$$ 2pt

(b) Sketch a graph of $f(x)$.

$f(x)$ is
x^2 moved RIGHT 6
& DOWN 2

2pt



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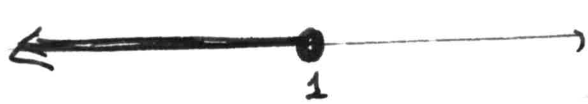
7. (a) [5 points] Find the domain of the following function.

$$f(x) = \sqrt{1-x}$$

Give your answer using interval notation.

Defined when $1 - x \geq 0$

when $1 \geq x$



1pt

$\boxed{\text{Domain is } (-\infty, 1]}$

3 pt for interval
1pt for brackets

(b) [5 points] Find the domain of the following function.

$$f(x) = \frac{\sqrt{2x+2}}{x-2}$$

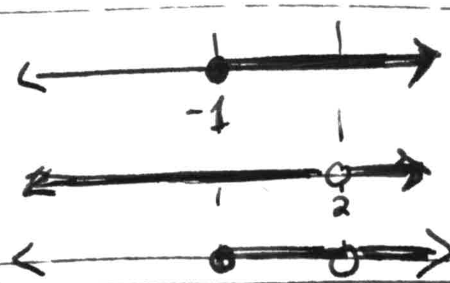
Give your answer using interval notation.

Defined when $2x+2 \geq 0$ AND $x - 2 \neq 0$

when $2x \geq -2$ AND $x \neq 2$

when $x \geq -1$ AND $x \neq 2$

in BOTH



1pt

AND

1pt

$\boxed{\text{Domain is } [-1, 2) \cup (2, \infty)}$

$\underbrace{}_{2pt}$

1pt for correct brackets.

7:07

8. Inverse Functions

(a) [5 points] Let $f(x) = 4x^3 + 2$. Find a formula for $f^{-1}(x)$.

find x s.t. $y = 4x^3 + 2$

$$y - 2 = 4x^3$$

$$\frac{y-2}{4} = x^3$$

$$x = \sqrt[3]{\frac{y-2}{4}} = f^{-1}(y)$$

$$\boxed{\underline{\text{so}} \quad f^{-1}(x) = \sqrt[3]{\frac{x-2}{4}}}$$

(b) [5 points] Suppose that f is defined using the following table:

x	1	3	5	9
f(x)	3	5	1	6

Compute $f^{-1}(3)$, $f^{-1}(1)$, and $f^{-1}(6)$.

$f^{-1}(3) = \underline{1}$ means $f(\underline{1}) = 3$

$f^{-1}(1) = \underline{5}$ means $f(\underline{5}) = 1$

$f^{-1}(6) = \underline{9}$ means $f(\underline{9}) = 6$

9. Polynomial Functions

(a) [8 points] Solve the following inequality. Give your answer in interval notation.

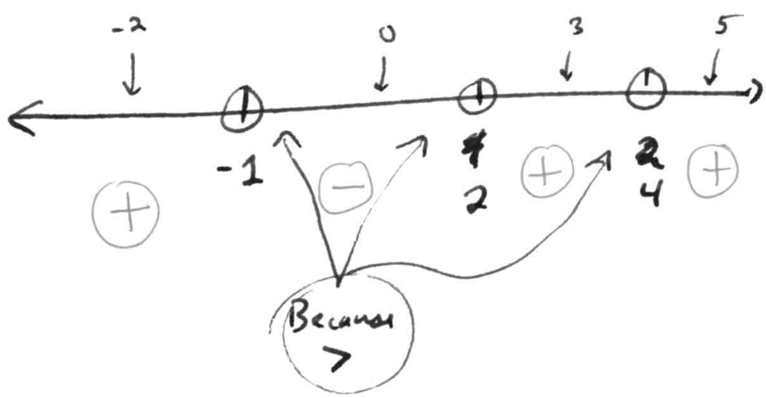
$$(x-4)^2(x+1)^3(x-2) > 0$$

(A) equality holds when

2pts for endpt

$x-4=0$ or $x+1=0$ or $x-2=0$

$x=4$ or $x=-1$ or $x=2$



(B) check intervals

$f(-2) = (\text{Neg})^2 \cdot (\text{Neg})^3 \cdot (\text{Neg})$

$= \text{pos} \cdot \text{Neg} \cdot \text{Neg}$

$= \text{pos}$

2pt for checking intervals.

$f(0) = \underbrace{(\text{Neg})^2}_{\text{pos}} \cdot (\text{Pos})^3 (\text{Neg}) = \text{Neg}$

$f(3) = (\text{Neg})^2 (\text{pos})^3 (\text{pos}) = \text{pos}$

$f(5) = (\text{pos})^2 (\text{pos})^3 (\text{pos}) = \text{pos}$

(C) Answer: inequality holds when x is in

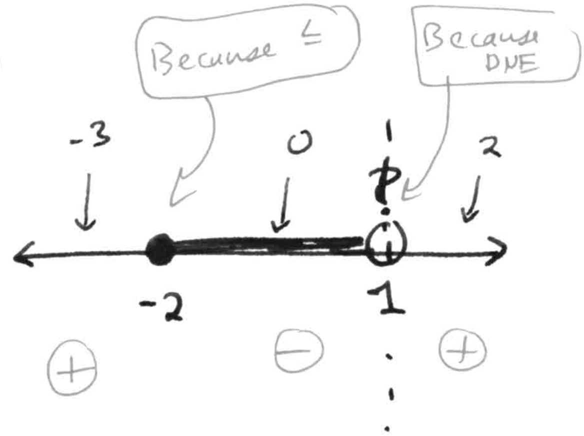
$$\boxed{(-\infty,-1) \cup (2,4) \cup (4,\infty)}$$

4 pts for answer (intervals & brackets)

10. Rational Functions

(a) [8 points] Solve the following inequality. Give your answer in interval notation.

$$\frac{2x+4}{x-1} \leq 0$$



Ⓐ equality when

$2x+4=0$

$2x=-4$

$x=-2$

2pt

Ⓑ DNE when $x-1=0$

$x=1$

Ⓒ check intervals

$f(-3) = \frac{-6+4}{-3-1} = \frac{-2}{-4}$ = pos

3pt

$f(0) = \frac{0+4}{0-1}$ = neg

$f(2) = \frac{4+4}{2-1}$ = pos

Ⓓ Answer

$\boxed{[-2, 1)}$

2pt interval

1pt brackets.

(b) [4 points] Let identify any horizontal and vertical asymptotes for the following function

$$f(x) = \frac{2x+4}{x-1}$$

vertical asymptotes

- can't simplify
- DNE when $x - 1 = 0$

when $x = 1$

$\Rightarrow$ vertical asymptote at $x = 1$

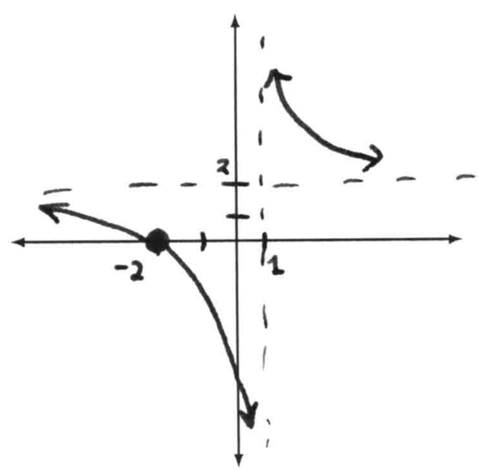
horizontal asymptotes

when x is big,

$f(x) \approx \frac{2\cancel{x}}{\cancel{x}} = 2$

$\Rightarrow$ horizontal asymptote at $y = 2$

(c) [2 points] Use your [redacted] work for (b) and (c) to sketch the graph of $f(x) = \frac{2x+4}{x-1}$



11. [8 points] Use polynomial division to rewrite $\dfrac{x^3+4x^2-x+2}{x^2+x+2}$ in the form $\dfrac{p}{d} = q + \dfrac{r}{d}$

$$\begin{array}{r}
x + 3 \\
x^2+x+2 \,\big)\, \overline{x^3 + 4x^2 - x + 2} \\
-(x^3 + x^2 + 2x) \\
\hline
3x^2 - 3x + 2 \\
-(3x^2 + 3x + 6) \\
\hline
-6x - 4
\end{array}$$

2pt ea step

$$x+3 + \frac{-6x-4}{x^2+x+2}$$

$$= x+3 + \frac{(-1)(6x+4)}{x^2+x+2}$$

$$= x+3 - \frac{6x+4}{x^2+x+2}$$

2pt for answer

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