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1 Algebra Review

1. Know the laws of exponents, fractions, and PEMDAS.
2. Proofs and counterexamples.
   (a) **Prove** that an equality **is always true** using laws of algebra.
   (b) Give a **counterexample** if an equality is **sometimes false**.
3. Be able to solve equations
   (a) Linear and Quadratic
   (b) Absolute value
   (c) Rational functions
4. Intervals
   (a) Be able to find the union and/or intersection of two or more intervals
   (b) Be able to translate between inequalities, number lines, words, and interval notation.

2 Function Concepts

1. Basic Concepts
   (a) The definition of a function
      i. The **Rule of Four:** Be able to define a function
         (1) verbally, (2) numerically, (3) algebraically, (4) graphically.
      ii. Modeling with functions.
         (1) Translating between *verbal descriptions* and *formulas*.
         (2) Keep track of input and output units.
      iii. Domain and Range
         (1) Know the definitions
         (2) Be able to find the domain and range of a function given numerically, graphically, verbally, and algebraically
   (b) Determining if something is a function
      i. Does each input give **only one output**?
      ii. Phrased graphically, this gives the vertical line test.
   (c) Evaluating a function
      i. Using a table
      ii. Using a formula

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iii. Using a graph

iv. Using a verbal description

(d) Solving equations and inequalities using the graph of a function.
(e.g. For what x does $f(x) = 4$? For what x is $f(x) \geq 4$?)

2. Linear Functions

(a) **Know and be able to use:** Formula for slope

(b) **Know and be able to use:** Slope-intercept and point-slope form for linear functions

(c) Modeling with linear functions

(For example, be able to interpret fixed and per-unit costs)

3. Piecewise Functions

(a) Be able to translate (both ways) between

i. the graph of a piecewise function

ii. the algebraic "formula" for a piecewise function

(b) Be able to evaluate a piecewise function at a given input.

4. Graphing Common Functions

(a) Linear functions

(b) Piecewise functions and absolute value functions.

(c) Know the graphs of x^2, $\sqrt{x}$, $|x|$, 2^x, $\left(\frac{1}{2}\right)^x$, e^x, and $\ln(x)$.

(d) Transformations of these common functions

i. Translating horizontally

ii. Translating vertically

iii. Stretching/compressing vertically

iv. reflecting across the x-axis

5. Composing Functions: Plugging one function into another.

(a) Composing two, three, or more functions.

(b) Decomposing Functions.

(c) Applications of composing (paying attention to physical meaning and units).

3 Inverses

1. The formal definition of "Inverse"

2. The concept behind inverses

"What do I need to plug into f to get it to output a certain value y?"

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3. Computing Inverses
   (a) Know the formal definition of f^{-1}.
   (b) Be able to evaluate f^{-1} when given f
   numerically, graphically, verbally, and algebraically
4. One-to-one functions
   (a) Idea: What is required for a function to have an inverse?
   (b) Does each output come from **only one input**?
   (c) Phrased graphically, this gives the horizontal line test.
5. Partial inverses
   (a) *Formally:* By restricting the domain of f to make it one-to-one,
   we can find a *partial inverse* of f
   (b) *Graphically:* If we cut off (erase) parts of a function f,
   we can find a *partial graph* that does pass the horizontal line test.
   Reflecting that partial graph across $y = x$ yields a partial inverse of f.

4 Synthesis: Interpreting Graphs

1. Be able to evaluate f at different inputs
2. Solve equations of functions like "find the x where $f(x) = 6$"
3. Solve equations with multiple functions like "find the x where $f(x) = g(x)$"
4. Solve inequalities like "find the x where $f(x) \geq 7$"
5. Read the Domain and Range off of a graph
6. Find the graph of f^{-1} from the graph of f.

5 Synthesis: Working with Functions

1. Use rules of algebra to simplify the composition of two or more functions.
2. Use knowledge of "when functions are defined" to find the domains of functions
   (a) Always: Find the inputs where the function is defined. It can help to start by looking for the places where it is **not** defined.
   (b) Know the rules for finding the domains of fractions, even roots, and compositions.
3. Use "Completing the Square" to graph and analyze quadratic equations.
   (a) Efficiently complete the square for quadratics with leading coefficient of 1.
   (b) Carefully complete the square for quadratics with leading coefficient other than 1.

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(c) Finding the vertex and graphing a given quadratic.

(d) Finding the maximum/minimum output attained by a function.
Finding the input where this output is attained.

4. Use knowledge of "solving equations" to find the inverse of a function given algebraically

   E.g. To find the inverse of $f(x) = x^3 - 5$, you solve $y = x^3 - 5$ for x in terms of y.

6 Polynomial Functions

1. The anatomy of a polynomial:

   Be able to identify the degree, leading coefficient, leading term, and constant term.

2. Polynomial Inequalities

   (a) Polynomials are smooth and continuous (no sharp corners, jumps, or holes)

   (b) By the *Intermediate Value Theorem*

   A continuous function f can only go between positive and negative at some input c where $f(c) = 0$.

   (c) To solve $f(x) > 0$ or $f(x) \leq 0$,

      i. Find the inputs c where $f(c) = 0$. Include/exclude as appropriate.

      ii. Make a sign chart and check intervals.

      iii. Read off answer.

3. Eventual Behaviour

   (a) When x is big, a polynomial "looks like" its leading term.

   (b) Be able to sketch x^n for n even, and be able to sketch x^n for n odd.

4. To Sketch a Polynomial

   (a) Sketch the leading term

   (b) Find where $f(x) = 0$

   (c) Find where f is positive and where it is negative.

   (d) Put it all together!

7 Rational Functions

1. *Rational functions* are fractions of polynomials.

   (a) Be able to identify if a given function is a rational function in disguise.

   (b) Know the graph of $f(x) = \dfrac{1}{x}$ and its translations.

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   (c) Know and be able to identify vertical and horizontal asymptotes.

2. Inequalities of Rational Functions

   (a) Rational functions are **not** continuous: they can jump and have holes

      A rational function f can only go between positive and negative at some input c where ***either*** $f(c) = 0$ ***or*** $f(c)$ DNE.

   (b) To solve $f(x) > 0$ or $f(x) \leq 0$,

      i. Find the inputs c where $f(c) = 0$. Include/exclude as appropriate.

      ii. Find the inputs c where $f(c)$ DNE. Always exclude.

      iii. Make a sign chart and check intervals.

      iv. Read off answer

3. Horizontal asymptotes and Eventual Behaviour

   (a) Know the definition of "horizontal asymptote"

   (b) When x is big, the top and bottom polynomials "look like" their leading term.

   (c) By simplifying the ratio of the leading terms, you can see where the function is "going"

4. Vertical Asymptotes and holes

5. To Sketch a Rational Function

   (a) Determine the eventual behavior
   This gives the horizontal asymptote, if it exists.

   (b) Determine the local behavior

      i. Find where $f(x) = 0$
      These are the function's zeros.

      ii. Find where $f(x)$ DNE
      These are the vertical asymptotes (or holes).

   (c) Find where f is positive and where it is negative.

   (d) Put it all together!

8 Polynomial Division

1. Be able to do it.

2. Know the ways to write out what $p(x)$ divided by $d(x)$ gives:

$$\frac{p(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

$$p(x) = q(x) \cdot d(x) + r(x)$$